Robust Portfolio Optimization using LSTM-based Stock and Cryptocurrency Price Prediction: An Application of Algorithmic Trading Strategies

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Abstract

Algorithmic trading (AT) has become widely used recently because of its high speed and accuracy in implementing diverse and complex strategies. Using algorithms also allows traders to execute their trading strategies in a high volume and numerous transactions without involving human emotions. While AT has many advantages, it also carries some risks due to the uncertain stock market conditions and the impact of news and political, social, and other events. Therefore, forming a stock portfolio and stabilizing against uncertainties, in conjunction with accurate market predictions, can significantly reduce risk.in this paper, For the first time, we developed a robust portfolio optimization model based on LSTM prediction using the AT strategies based on short-term moving average techniques. First, we implement the strategies derived from the VLMA, FLMA, EMA, and SMA algorithms based on the LSTM's predicted price. Secondly, we develop a robust portfolio optimization model using the abovementioned algorithms. The results show that in both stock and crypto portfolios, moving average strategies will perform better than the benchmark strategy (Buy-and-hold). Also, when the model parameters are deterministic, the robust portfolio constructed stocks and crypto will perform better than Buy-and-hold for all algorithms. However, when the variance from certain models increases, VLMA and FLMA (15-day holding) for stocks and FLMA (30-day holding) for the crypto will not be a suitable investment option. Additionally, portfolios constructed using all AT strategies and all assets outperform the benchmark portfolio in certain and non-certain markets.

Keywords: Robust portfolio optimization, Algorithmic trading, Price prediction, LSTM

1. Introduction

In the past few years, the financial industry has experienced significant progress and growth thanks to the rapid development of computer technology. This has paved the way for exciting and valuable opportunities for market participants, with algorithmic trading (AT) being a noteworthy tool. AT, or algo-trading or automated trading, is a computerized trading system that executes trades based on pre-programmed instructions. Furthermore, it has reduced trading costs, minimized the impact of human emotions on trading decisions, and increased market liquidity, which has made AT popular with many investors and researchers.

Providing a situation with an accurate stock market prediction with the help of machine learning and deep learning algorithms is one of the other advantages of developing computer technologies in finance. These advanced algorithms analyze vast amounts of historical data and use it to identify patterns and trends that can help predict future market movements. This can be a precious tool for investors and financial professionals looking to make informed decisions about their investments. By leveraging the power of machine learning and deep learning, finance professionals can gain a deeper understanding of market trends and make more accurate predictions of the stock market.

While the stock market offers investors access to innovative investment tools and methods that can yield significant benefits, it is susceptible to fluctuations triggered by dynamic variables such as breaking news, economic developments, and socio-political events like the COVID-19 pandemic and the Russia-Ukraine conflict. These fluctuations increase the risk associated with investments. Thus, forming a well-diversified portfolio that is robust enough is essential to mitigate these risks and increase investment return. For the first time, Markowitz (1952) combined the risk and return concepts in the mean-variance model.

One particularly well-regarded method that has captured the attention of researchers is the LSTM method, known for its accurate forecasting and other associated benefits (Graves and Schmidhuber (2005), Ta et al. (2020), Rather (2021), Cipiloglu Yildiz and Yildiz (2022)). Moreover, the adoption of technical analysis strategies in algorithmic and applied trading has emerged as an attractive area for researchers and investors (Frattini et al. (2022), Aycel and Santur (2022), Kalariya et al. (2022)). It is worth noting, however, that the implementation of algorithmic trading strategies based on predicted prices with LSTM has not been widely observed, potentially leading to significant losses. Another critical issue is the underutilization of uncertainty layers to manage uncertainties in portfolios based on LSTM.

The principal objective of this study is to propose a robust portfolio optimization model grounded in prediction established through the algorithmic trading approach. To address the research inquiries, this investigation introduces a three-phase model. The process begins with using the LSTM model to predict stock prices. Then, algorithmic trading techniques are used to implement strategies based on moving averages. After that, the minimum and maximum return values for each trading algorithm are determined. In the final stage, a strong portfolio is created for each trading algorithm, and then a diversified, robust portfolio is formed based on all the algorithms. The results show that this approach delivers better performance in stock and crypto portfolios than the benchmark strategy. Additionally, the combined robust portfolio outperforms the buy-and-hold strategy across all levels of uncertainty. The remainder of this paper is organized as follows. Section 2 provides a concise overview of recent AT methods through a literature review. Section 3 is devoted to mathematical models and strategies. Section 4 presents the experiments performed and the computational results. Section 5 is devoted to the discussion and managerial insights, and Section 6 concludes the paper and provides suggestions for possible future research.

2. Literature survey

2.1. Prediction

Price prediction plays a significant role in asset allocation, risk management, and asset valuation (Dai and Zhu, 2020). Therefore, many researchers have focused on this topic in recent years. In this article, we will analyze a few of them. Ariyo et al. (2014) in their article presented a process for building an ARIMA model for forecasting stock prices on the NYSE and Nigeria Stock Exchange (NSE). Furthermore, the article asserts that the mentioned model provides better prediction power in the short term than existing methods. Siami-Namini et al. (2019) compare the behavior of LSTM and BILST to discover how useful additional layers of trained data are for adjusting the involved parameters. Finally, they conclude that BILST-based models have better forecasting performance than ARIMA and LSTM-based models as a result of additional data training. Shahvaroughi Farahani (2021) attempted to forecast interest rates by employing Artificial Neural Networks (ANN) and enhancing the network's performance by utilizing innovative heuristic algorithms such as MFO, CHOA, and TVAC-PSO, among others. he entered 17 variables, including oil, gold coin, and house values. Sapankevych and Sankar (2009) use the SVM model to predict stock prices as this method is highly effective in predicting nonlinear, non-stationary, and undefined a priori time series. Mondal et al. (2014) Analyzed the predictive power of the ARIMA model on 56 stocks from different industries in the Indian stock market. The Akaike Information Criterion (AIC) was used in this research to compare and parameterize the ARIMA model. Chen et al. (2021) introduced a novel portfolio construction approach that uses a hybrid machine learning and mean-variance model. As part of the hybrid model, stock prices are predicted using Extreme Gradient Boosting and IFA, and portfolios are selected using the MV model, which selects stocks with higher potential returns. According to a study on the Shanghai Stock Exchange, the method outperforms traditional methods and benchmarks regarding returns and risks. Sen et al. (2021) present the creation of optimal risk and portfolios in India's four economic sectors using ten significant stocks from web-extracted prices between 2016 and 2020. The paper also evaluates the performance of an LSTM model in forecasting stock prices. It compares the predicted and actual returns six months after the portfolios were constructed. LSTM model performs accurately, demonstrating its capability to forecast stock prices in the Indian market. Imajo et al. (2021) presented two techniques for improving financial prediction algorithms. The first is a technique for efficiently extracting information that can be combined with different prediction models. Second, a novel neural network architecture incorporates biases associated with financial induction, such as fractal dimensions and time scale invariance. U.S. and Japanese stock market data are used to demonstrate their effectiveness. Experimental ablation confirms their contributions. Fu and Wang (2020) developed a multi-period portfolio model that utilizes predictive data to allocate resources efficiently. To optimize resource allocation and predict future stock prices, they employ LSTM neural networks and particle swarm optimization algorithms. The approach they have developed has been validated by empirical research. Ferdiansyah et al. (2019) studied using LSTM modules for Bitcoin forecasting. These modules have gained popularity among researchers and have the same recurrent properties as RNNs. Based on the research, Bitcoin on Yahoo Finance will surpass USD 12600 in the days following the prediction. In Schöneburg (1990), neural networks were used to predict short-term stock prices for three randomly selected German stocks (BASF, COMMERZBANK, and MERCEDES). Within a 10-day prediction period, the researcher achieved promising results with accuracy rates up to 90% using PERCEPTRON, ADALINE, MADALINE, and BACK-PROPAGATION networks. The BACK-PROPAGATION network demonstrated behavior similar to exponential smoothing. These findings suggest that Stock price prediction could be enhanced by neural networks. Obthong et al. (2020) examined the challenges associated with stock price forecasting and the importance of accurate information for effective decision-making. Various methods and algorithms for improving accuracy in stock price prediction are examined in this review. To optimize the problem of stock price prediction, Lee (2001) proposed a reinforcement learning model as a Markov process. As a result of experimental testing on the Korean stock market, this method demonstrated promising performance in improving accuracy in stock price prediction. Although, investors can gain enormous benefits from the novel investment tools and methods that are available in the stock market. However, considering its nature and the high effectiveness of news, economic parameters change, and political and social events, such as the dollar's removal from global exchanges, the coronavirus epidemic, the Ukraine-Russia war, etc. Investing in the stock market can be risky. Therefore, it is necessary to form a stock portfolio. Novel mean-variance portfolio optimization model that balances risk and return, Presented by Markowitz. By using this model, the investor can create his ideal portfolio based on his risk tolerance. Several studies have been conducted to complete and improve the Markowitz model. A mean-variance portfolio selection model with stochastic parameters published by Lim and Zhou (2002) focuses on the Continuousmean-variance Portfolio selection model with random interest rate and volatility coefficients in a complete market. This problem is solved by using the optimal quadratic linear equation and backward differential equations. Through this quadratic linear equation, the strategies, as well as the efficient frontier of the average variance, are extracted. Low et al. (2016) attempt to answer why mean-variance models work poorly. In response to this question, first, by sampling the multivariate probability model that explicitly includes distributional asymmetry, it estimates the expected return. The study also demonstrates that marginal models of dynamic characteristics, such as volatility clustering and skewness, are effective in reducing estimation errors compared to historical sampling windows. Finally, several models based on average variance exhibit significant statistical performance even after accounting for transaction costs. Due to its simplicity, accessibility, and robustness, mean-variance portfolio theory is usually used in investments. However, one of the gaps in this theory is ignoring the non-Gaussian returns of higher moments Lassance (2022) Provided the updated mean-variance portfolio using non-Gaussian returns, it optimizes the selected high moment criteria. In multi-objective optimization, convergence and diversity are two essential aspects. Researchers have made numerous attempts to design convergent systems in recent decades.

2.2. Algorithmic trading

Moldovan et al. (2011) proposed a trading algorithm that uses multiple technical indicators to increase the accuracy of trading signals. The indicator parameters can be adjusted quickly and reliably with this solution but can also be used to identify new trading rules. Trading rules discovered automatically and by experts' observations could be combined to create an automated trading system. Risk-return balances must be controlled since everyone has different risk tolerance levels. This may lead to the development of evolutionary trading systems. Lei et al. (2020) improved the efficiency of the traditional MACD algorithm based on residual networks. They believe deep learning networks can analyze the likelihood of a trading point succeeding based on market behavior. The MACD-KURT strategy was tested on the Chinese market based on Residual Networks predictions and analysis of technical indicators. According to the results, a strategy employing residual network forecasting and indicators analysis together is more effective compared to a strategy based exclusively on technical analysis. This is both in terms of risk and returns. An agent-based deep reinforcement learning framework is presented by Shavandi and Khedmati (2022) that utilizes the collective intelligence of multiple agents. It works in a hierarchical structure where knowledge flows from high-timeframe agents to low-timeframe agents. This makes them highly robust to financial time series noise. Results indicate that thedeveloped multi-agent framework, based on several return-based and risk-adjusted performance measures, outperforms individual agents and several benchmark trading strategies in all investigated trading timeframes. Multi-agent frameworks are Appropriate for AT in financial markets because of their robust performance. Scholtus et al. (2014) found that news-based trading strategies become significantly less profitable if there is a delay of 300 MS or more in news delivery. The reduction in volatility is more pronounced on days with Efficacious news and high levels of volatility. They also examine AT's impact on market quality around the release of macroeconomic news. It was stated in their report that automated activity increases trading volume and depth within one minute of the arrival of macroeconomic news. A real-world setting where prices are interrelated is considered by Iqbal et al. (2019). In this scenario, each price is determined by the price that precedes it. They also derive a lower bound on non-preemptive randomized algorithms. Based on the erroneous and fixed price bounds, they developed anUpdated model that enhances the bounds. Based on theupdated model, they proposed a non-preemptive reservation price algorithm RP* and analyzed it using a comparative analysis framework. A quantitative analysis of time series data was used to propose trading strategies by Salkar et al. (2021) to achieve high profits on intraday trades, and these strategies were developed. Findings indicate that the strategy combining RSI and MACD provides the highest returns of up to 12%. Chang et al. (2017) assessed the performance of every stock listed on theTaiwan Stock

Exchange (TWSE) using variable-length moving averages (VMAs). They calculated the excess returns of technical trading in comparison with BH trading. In addition, the study results indicate that VMAs perform better than BH strategies. Furthermore, the profitability of VMAs is positively correlated with the size and volume of trades. Pradhan et al. (2021) developed the strategy of $SMA(30,100)$ based on the predicted price by LSTM.

Garcia and Schweitzer (2015) analyzed economic and social signals about Bitcoin prices. These signals included exchange volume, technology adoption, Analysis of Bit coin-related tweets for information, word-of-mouth volume, emotional valence, and opinion polarization. The findings of the study inform profitable algorithmic trading strategies for Bitcoin, which leverage the sentiments expressed in social media to generate positive investment returns. Vo and Yost-Bremm's (2020) Applied Design Science Study to develop a high-frequency trading strategy for Bitcoin based on six exchanges as their Information Technology artifact. Their strategy incorporates indicators related to the financial market and algorithms based on machine learning. Chaboud et al. (2014) examined AT and its effects on the FOREX utilizing extensive high-frequency data. They observe that AT improves price efficiency by reducing triangular arbitrage opportunities and decreasing autocorrelation of high-frequency returns. Hendershott et al. (2011), studied the effect of AT on liquidity by employing the NY Stock Exchange (NYSE) automatic quote dissemination system in 2003 as an external control. AT decreases spreads, minimizes adverse choices, and diminishes trade-related pricing, especially in large firms. The results of this study indicate that AT contributes to improved liquidity and informativeness of quotes. Weller (2017) has challenged the idea that algorithmic trading can enhance price efficiency. Based on ananalysis of stock-quarter data provided by the Securities and Exchange Commission (SEC), researchers found that the amount of information in prices diminished between 9% and 13% per standard deviation of AT activities one month before the planned disclosure. Another innovative tool available to investors, investment funds, and other institutions is the ability to predict market fluctuations with high accuracy through advanced methods and techniques. Machine learning, artificial neural networks, deep learning, etc., are among the methods and techniques that are involved in these processes.

2.3.Robust portfolio optimization

A robust portfolio optimization process considers the worst-case scenario and performs optimization based on that scenario. Robust optimization for the first time waspresented by Soyster (1973) in the form of a linear optimization model that provides the best-justified solution regardless of the duration of the input data. This approach produces conservative results in practice. To ensure robustness, we move away from the optimality of the nominal problem. This field has been the subject of numerous research studies. Bertsimas and Sim (2004) presented a model that can be adjusted to resolve high conservatism. This can only be done by setting one parameter. This model's linearity and ability to solve integer models quickly. Following, we will briefly review some examples of robust portfolio optimization models. In this section, we will review a few examples. Goldfarb and Iyengar (2003) Formulated and solved a robust portfolio selection problem (allocation). An optimal portfolio is analyzed in this study to determine its sensitivity to errors as well as the optimization of relevant parameters of the market. According to the study's findings, uncertainty structures are related to statistical confidence regions used to evaluate parameters. Eskorouchi et al. (2023) conducted a bibliometric analysis to investigate recent progress in the robustness of portfolio optimization in light of the current global economic instability impacting financial markets. Kim et al. (2018) described recent developments in the classification of robust optimization models, the allocation of assets according to asset classes, and the selection of portfolios based on private assets. Additionally, robust portfolio selection methods suitable for each asset category were separated. A hierarchical model for robust investment between two risky assets has been proposed by Lin et al. (2022). The model consists of two steps. First, it chooses a relatively safe asset, and then it decides how much to invest in the relatively risky asset to avoid uncertainty in the relatively safe asset. A robust portfolio optimization model based on evidence theory was presented by Eskorouchi et al. (2022); using the Dempster-Shaffer model, they determined each share's range. Following this, the Bertsimas model was applied to calculate stock portfolio returns at different levels of uncertainty. The literature review indicates that there are two gaps in algorithm trading studies. First is the lack of integration between AT studies and robust portfolio optimization models. Secondly, a study that combined the VLMA, FLMA, EMA, and SMA algorithms with the predicted price using LSTM in ashort-term period was not observed.

Therefore, in this paper, to cover the mentioned gaps, for the first time we developed a robust portfolio optimization model based on LSTM prediction using AT algorithm. First, we predicted daily price fluctuations using the LSTM model. In the next step, for each of the VLMA, FLMA, EMA, and SMA algorithms based on thepredicted price we developed a robust portfolio optimization model. This study was conducted between January 2018 and July 2023 on ten assets, including five shares of the American stock market and five cryptocurrencies.

3. Model

In this section, we present the methodology employed in our study, which consists of three key subsections: portfolio optimization model, LSTM prediction model, and algorithmic trading strategies. Each subsection focuses on a specific aspect of our research and contributes to the overall framework of our robust portfolio optimization using LSTM-based stock and cryptocurrency price prediction.

3.1. Mean-Variance Portfolio Optimization Model (MV)

Mean-variance Portfolio optimization was introduced by (Markowitz, 1952). The objective of this model is to balance profit and risk. The first objective function in Eq (1) focuses on minimizing portfolio risk, whereas Eq (2) is geared towards maximizing returns. Eq (3) is a budget constraint, indicating that sum of all stock weights must equal 1. It is also possible to define restrictions based on different conditions. Markowitz's model takes the general form shown below:

$$min \sum_{i,j=1}^{n} x_i \times x_j \times cov_{i,j} \tag{1}$$

$$max \sum_{i,j=1}^{n} r_i \times x_j \tag{2}$$

$$S.T. \sum_{i=1}^{n} x_i = 1 \tag{3}$$

$$x_i \geq 0, \qquad \forall i \in (1, \dots, n) \tag{4}$$

the variable x denotes the allocated budget for each stock, while n represents the total number of stocks in the portfolio and r is the stock's historical return. Eq (4) explicitly dictates the prohibition of short selling.

3.1.1. MVF model.

To develop the Markowitz Mean-Variance model (MV), Yu et al. (2020) developed a model called MVF. Unlike MV which relies on historical data, the MVF model uses the predicted rate of return for its calculations, (Ma et al., 2021). In addition to minimizing risk and maximizing return, it provides a novel objective function for reducing prediction error eq (7).

$$Min \sum_{i,j=1}^{n} x_i \times x_j \times cov_{i,j} \tag{5}$$

$$Max \sum_{i,=1}^{n} x_i \times r\hat{}_i \tag{6}$$

$$Max \sum_{i=1}^{n} x_i \times \varepsilon_i \tag{7}$$

$$S.T. \sum_{i,=1}^{n} x_i = 1 \tag{8}$$

$$x_i \geq 0, \qquad \forall i \in (1, \dots, n) \tag{9}$$

The above model uses $r\hat{}_i$ instead r_i that is predicted return, and also $\varepsilon_i = r_i - r\hat{}_i$ calculates prediction errors.

3.1.2. case study: Robust MVF model

This analysis, similar to the MVF, computes the rate of return using forecast data. However, it employs an uncertainty interval return based on algorithmic trading strategies. Therefore, this paper assumes that the return is an interval uncertainty parameter and utilizes Bertsimas' robust optimization to address these uncertainties. eq (10) Shows a uniform interval of changes of predicted return ($\tilde{r}$).

$$\tilde{r} \sim (\tilde{r} - \hat{a}_i \,, \tilde{r} + \hat{a}_i) \tag{10}$$

In this study, to maintain the linearity of the model we focus on the first part of the objective function of the model and run the robust model on it specifies the level of conservatism and determines how many parameters can have their maximum value. eq (15) specifies that the maximum weight assigned to each AT strategy in portfolio should be 40% of the total budget and improves portfolio diversification

$$Max\ U \tag{11}$$

Subjected to:

$$U - \sum_{i=1}^{n} \hat{r}_{i,k} \times x_{i,k} + z_j \Gamma_j + \sum_{i} p_{ij} \leq 0 \quad \forall\ j \tag{12}$$

$$z_j + p_{ij} \geq a_{ij}\ y_{i,}\ , \qquad \forall\ i,j \in j_j \tag{13}$$

$$-y_i \leq x_{i,k} \leq y_i \qquad \forall\ i,k \tag{14}$$

$$x_{i,k} \leq 0.4 \qquad \forall\ i,k \tag{15}$$

$$x_{i,k} \geq 0 \qquad \forall\ i,k$$

$$p_{ij} \geq 0, \quad \forall\ i,j \in j_j \tag{16}$$

$$y_i \geq 0, \ \forall\ i \tag{17}$$

$$z_j \geq 0\ ,\ \forall j \tag{18}$$

In this study, $\boldsymbol{i}$ denotes the index associated with each asset. Furthermore, $\boldsymbol{k}$ is the implemented AT strategies index on each asset, including FLMA (15), SMA, FLMA (30), EMA, and VLMA. $\hat{r}_{i,k}$ is the uncertainty parameter of the model, it shows the expected return for implementing each AT strategy on each asset. $x_{i,k}$ shows the dedicated weight when applying each AT strategy on each asset. Γ specifies the uncertainty level, and $\hat{a}_{i,k}$ shows the maximum possible variance from $\hat{r}_{i,k}$.

3.2. LSTM prediction model

As the name suggests, LSTM stands for Long-Short-Term-Memory. It is a type of RNN network. However, it has long-term memory, while RNN does not have this possibility. There are generally three layers in an LSTM neural network: the input layer, the hidden layer, and the output layer.

3.3. Algorithm trading strategies

Buy and hold strategy

Buy and hold is the oldest and simplest trading strategy. Likewise, in this article, it has been used as a benchmark strategy by many researchers, including (Mohr et al., 2014) and (Chang et al., 2017). This strategy generates the buy signal on the first day of the study period, and the sell signal on the last day of the period.

Simple Moving Average (SMA)

Moving average is considered one of the oldest and most popular technical analysis tools, and it has different types. As the name suggests, the Simple Moving Average (SMA) is the simplest type of the Moving Average family. If $\boldsymbol{MA_{short} > MA_{long}}$, a buy signal is issued. And when $\boldsymbol{MA_{short} < MA_{long}}$ a sell signal is issued. Algorithm 1 presents a formal description of the algorithm.

Table 1. Structure of SMA

Algorithm 1: Simple Moving Average Algorithm (SMA)
REQUIRE: MA_{long} , MA_{short}
1. if $MA_{short} > MA_{long}$
2. A buy signal should be generated
3. end if
4. if $MA_{short} < MA_{long}$
5. A sell should be generated
6. end if
7. Issue a sell signal on the last trading day, regardless of the open position

Exponential Moving Average (EMA)

An exponential moving average (EMA) is based on generating buy and sell signals in the same manner as a simple moving average (SMA) in Algorithm 1. But in the roll's calculations $eq(19)$, act differently. In this technique, more weight is given to the recent day's value.

$$EMA_{t1} = price_{t1} * \left(\frac{smoothing}{1 + days}\right) + EMA_{t0} * \left(1 - \left(\frac{smoothing}{1 + days}\right)\right) \tag{19}$$

Variable and fixed length moving average: (VLMA) & (FLMA)

To prevent multiple signals from being generated in SMA, (Brock et al., 1992) introduced a new model that was used by (Ahmad et al., 2021), (Zhu et al., 2015), and (Ming-Ming & Siok-Hwa, 2006). this algorithm generates a buy signal when $\boldsymbol{MA_{short} > (1+\beta) * MA_{long}}$ and the sell signal when **if** $\boldsymbol{MA_{short} < (1-\beta) * MA_{long}}$The FLMA model can also be operated similarly, except that in this case we also have a maintenance period. In other words, once a signal has been generated, the position must be held for a fixed period. For example, if the holding time is 30 days, the current open position will be maintained for those 30 days, regardless of the type of new signal issued.

Table 2. Structure of (VLMA) & (FLMA)

Algorithm 2: Variable Moving Average Algorithm (VLMA) & Fixed Moving Average Algorithm (FLMA)
REQUIRE: $Band(\beta)$ MA_{long} ,MA_{short}

1. if $MA_{short} > (1+\beta) * MA_{long}$
    2. A buy signal should be generated
        3. end if
4. if $MA_{short} < (1-\beta) * MA_{long}$
    5. A sell should be generated
        6. end if
7. Issue a sell signal on the last trading day, regardless of the open position

4. *Experimental Results*

First, we extracted daily price data for the five largest shares of the American stock market, including APLE, TSLA, AMD, GOOG, SONY, and five cryptocurrencies from the crypto market, BITCOIN, Dodge, ADA, BNB and ETH, from the finance.yahoo.com, From January 2018 to June 2023. The data from 2018 to November 2022 are considered training data, and then using the LSTM neural network model (kaggle.com), we predicted the six months of December 2022 to June 2023 for each asset. Table 3 displays mean squared error (MSE) , the root mean squared error (RMSE), and mean absolute error (MAE) metrics for LSTM based on the change in return percentage over the past 45 trading days. (20), (21), and (22) equations are, MSE, RMSE, and MAE metrics calculations.

$$\mathbf{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\hat{r}_i - r_i)}{r_i}\right)^2 \quad (20) \qquad \mathbf{MAE} = \frac{1}{n}\sum_{i=1}^{n}|\hat{r}_i - r_i| \quad (21) \qquad \mathbf{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\hat{r}_i - r_i)}{r_i}\right)^2} \quad (22)$$

The effectiveness of the LSTM model in ranking is demonstrated using the Rank Graduation Accuracy (RGA) measure, as shown in Equation (23), and Spearman's Rank Correlation Coefficient (ρ) in Equation (24), with the results presented in Table 3. In these equations, $Y_{\hat{r}}(Y)$ denotes the actual price reordered by the predicted ranks, Y represents the actual price of assets, and F(Y) is the cumulative distribution function based on the actual price, (Raffinetti, 2023). Additionally, d_i represents the squared difference between the predicted and actual ranks, and q is for days in the test period, (Spearman, 1961). According to the results in Table 3, the MSE and RMSE criteria indicate that Apple and Sony have the lowest prediction errors. Additionally, Apple shows the smallest variance from the actual value based on the MAE criterion, with an error of 1.19E-02. Overall, the LSTM model demonstrates high predictive accuracy, with errors below 4% across all three criteria and for all assets studied.

The RGA criterion, which evaluates the model's effectiveness in predicting the relative ranking of assets, achieves the highest accuracy for ADA, with a value of 0.95. In contrast, the model's accuracy in ranking assets such as SONY (0.57) and APLE (70.61%) is comparatively lower. Additionally, the LSTM model exhibits a strong positive rank correlation (ρ) between actual and predicted values across most assets. BTC

$$\mathbf{RGA} = \frac{cov(Y_{\hat{r}}(Y), F(Y)) + cov(Y, F(Y))}{cov(Y, F(Y)) * 2} \quad (23) \qquad \boldsymbol{\rho} = \frac{6 * \sum_{i=1}^{n} d^2{}_i}{q(q^2-1)} \quad (24)$$

Table 3. Comparing prediction accuracy with various measurements

Metrics	Stock					Cryptocurrency				
	tesla	aple	sony	amd	goog	btc	ada	bnb	eth	doge
MSE	8.27E-04	2.30E-04	3.53E-04	1.46E-03	3.21E-04	1.09E-03	8.39E-04	8.45E-04	6.13E-04	8.89E-04
RMSE	2.88E-02	1.52E-02	1.88E-02	3.83E-02	1.79E-02	3.30E-02	2.90E-02	2.91E-02	2.48E-02	2.98E-02
MAE	2.17E-02	1.19E-02	1.42E-02	2.88E-02	1.39E-02	2.42E-02	3.47E-02	2.08E-02	1.92E-02	2.09E-02
RGA	%86.96	%70.61	%57.69	%91.03	%81.92	%80.08	%95.03	%71.28	%78.53	%79.01
ρ	%95.66	%88.73	%95.02	%96.29	%94.80	%97.61	%97.45	%93.21	%96.50	%95.23

shows a particularly high correlation of 97.61%, while ETH achieves a correlation of 96.50%, indicating a strong alignment with actual values. The results for the RGA and ρ criteria indicate that the LSTM model demonstrates a strong ability to detect the general direction of asset movements. This capability can aid in identifying suitable strategies for algorithmic trading and enhance the model's robustness against future market fluctuations. Then, we implemented short-term trading strategies derived from FLAMA (15) FLAMA (30), VLMA, EMA, SMA algorithms on predicted price. Table 4 shows the results of each strategy.

Which strategy is most profitable?

In this study, as well as in (Schmidt et al., 2010), returns have been calculated utilizing the geometric average trading period return (GPR) in equation (25).

Table 4. AT Strategies geometric rate of return

srategy	*Hold time*	*stock portfolio*	*crypto portfolio*	*(stock + crypto)*
FLMA (5,15,0.01)		1.086	1.081	1.083
FLMA (6,18,0.01)		1.061	1.094	1.078
FLMA (10,30,0.01)		1.024	1.078	1.051
FLMA (15,45,0.01)	15	0.995	0.988	0.991
FLMA (5,15,0.02	days	1.057	1.103	1.080
FLMA (6,18,0.02)		1.030	1.085	1.057
FLMA (10,30,0.02)		1.042	1.033	1.038
FLMA (15,45,0.02)		0.961	1.026	0.993
VLMA (5,15,0.01)		1.085	1.0439	1.064
VLMA (6,18,0.01)	0	1.063	1.0408	1.052
VLMA (10,30,0.01)		1.031	1.0549	1.043
VLMA (15,45,0.01)	day	0.987	0.9881	0.988
VLMA (5,15,0.02		1.059	1.0472	1.053
VLMA (6,18,0.02)		1.039	1.0316	1.035
VLMA (10,30,0.02)		1.042	1.0335	1.038
VLMA (15,45,0.02)		0.961	1.0254	0.993
FLMA (5,15,0.01)		1.117	1.124	1.120
FLMA (6,18,0.01)		1.098	1.126	1.112
FLMA (10,30,0.01)	30	1.054	1.062	1.058
FLMA (15,45,0.01)		0.991	0.980	0.986
FLMA (5,15,0.02	days	1.100	1.110	1.105
FLMA (6,18,0.02)		1.079	1.090	1.085
FLMA (10,30,0.02)		1.046	1.020	1.033
FLMA (15,45,0.02)		0.967	1.019	0.993
SMA (5,15)		1.119	1.036	1.077
SMA (6,18)		1.094	1.047	1.071
SMA (10,30)	**	1.014	1.042	1.028
SMA (15,45)		1.003	1.017	1.010
EMA (5,15)		1.085	1.045	1.065
EMA (6,18)	**	1.066	1.043	1.054
EMA (10,30)		1.058	1.071	1.065
EMA (15,45)		0.945	1.100	1.039
B&H (5)		1.007	1.001	1.004
B&H (15)		1.017	1.010	1.014
B&H (30)	**	1.033	1.032	1.033
B&H (45)		1.030	1.038	1.034
GPR		1.039	1.045	1.044

$$\mathrm{GPR} = \left(\prod_{i=1}^{n} r_i\right)^{1/n} \quad (25)$$

Results indicate that crypto portfolio with GPR 1.045 will have a higher profit than the stock portfolio with GPR 1.039.

In the ***first portfolio (stocks)***, SMA (5,15) with 1.119 and $FLMA$(5,15,0.01) with 1.117 will have the highest GPR. As compared to the FLMA algorithm with a holding period of 15 days, $FLMA$ (5,15,0.01) has the highest return of 1.086. Also, for the VLMA algorithm, $VLMA$ (5, 15, 0.01) has the highest return. Additionally, $FLMA$ (5,15,0.01) is the best strategy for the FLMA algorithm with a 30-day holding period and GPR 1.117. According to the SMA algorithm, SMA (5,15) is the most profitable, whileEMA(5,15) will be the best strategy with GPR 1.085.

For the ***second portfolio (cryptocurrencies),*** $FLMA$(6,18,0.01) in 30 days is the best strategy with a GPR of 1.126. Based on the FLMA algorithm with 15-day holding time, $FLMA$ (6,18,0.02) exhibits the highest performance. Among the VLMA algorithms, $VLMA$(10,30,0.01) will have the highest GPR at 1.054. Also SMA(6,18) strategy with GPR 1.047 for the SMA algorithm and EMA(15,45) will have the greatest performance in the EMA algorithm for the second portfolio with a GPR value of 1.1.

Develop a robust portfolio optimization model

Considering that the GPR calculated in table 4 is based on the predicted price, and the prediction will always include some errors, it is impossible to formulate definitive conclusions about the market. Therefore, to manage these uncertainties, we developed a robust portfolio optimization model for each portfolio. Figure 1 presents the performance of the different algorithms for Uncertainty levels in the robust portfolio. Based on the results shown in this figure, when we are in stable conditions, the FLMA algorithm with a 30-day holding time as a result, it will be the most suitable algorithm for investment in a stock portfolio. Also, by increasing variance from the central limit (a highly volatile situation) for the first portfolio, the EMA algorithm will perform better than all the other algorithms. In addition, for the crypto portfolio EMA algorithm performs better than the others for certain uncertainty conditions.

Furthermore, Figure 2 illustrates which portfolio is most suitable for each investment algorithm. According to the SAMA algorithm, the first portfolio (stocks) is expected to have a higher profit. Also, the FLMA algorithm with a 15-day holding period for the crypto portfolio and the FLMA algorithm with a 30-day holding period for the stock portfolio would be better investment options. In stable market conditions, VLMA performs better on the stock portfolio. However, this algorithm will be more beneficial to the crypto portfolio when market uncertainty increases. Although the crypto portfolio is more advantageous in a deterministic state, when uncertainty is high, the algorithm produces the same results on both portfolios.

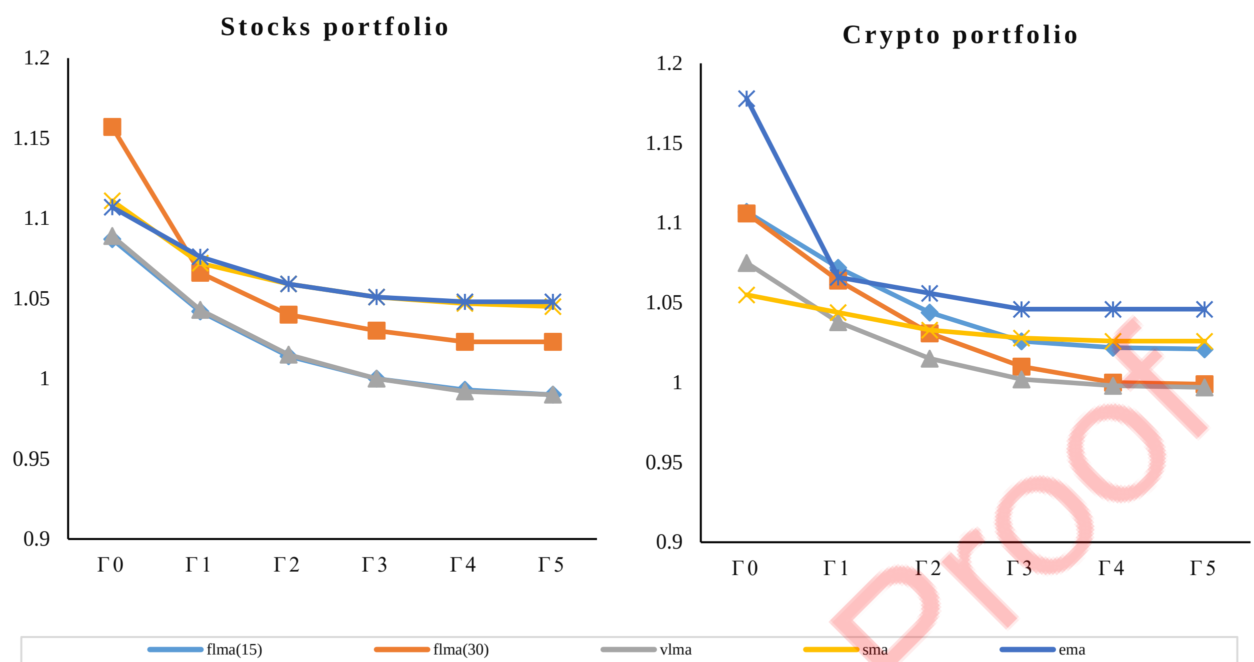

Figure 1. Analyzing the Performance of Algorithms in Stock and Cryptocurrency Portfolios

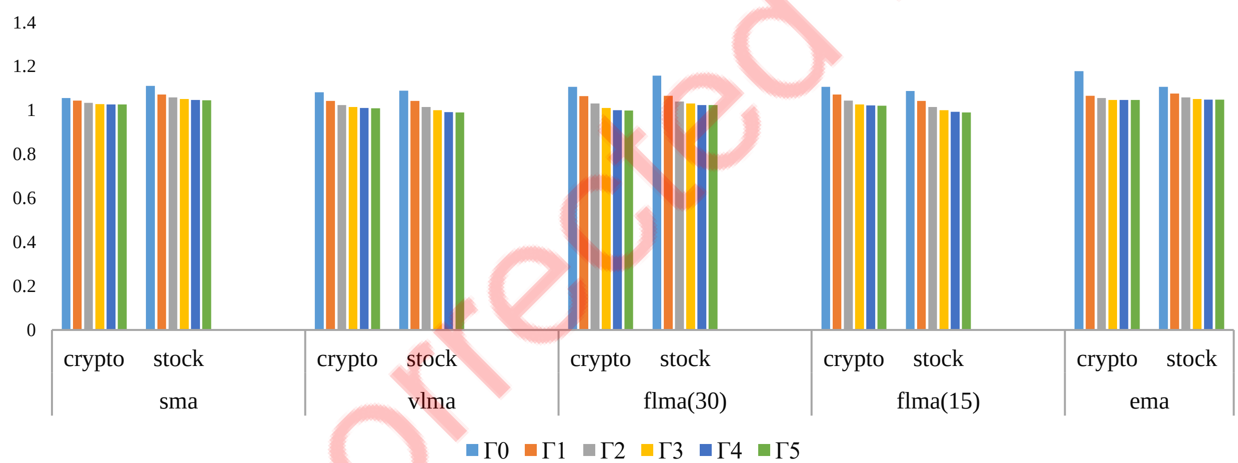

Figure 2. Comparing the profitability of stock and crypto portfolios using different algorithms.

According to Figure 3, which shows the excess return of each algorithm over the benchmark algorithm (B&H) for both portfolios, results show that, for the first portfolio, all the moving average algorithms outperform the benchmark algorithm when the model is certain. However, in situations of increased uncertainty (most variance from certain conditions), the VLMA and FLMA (15-day holding) algorithms will not be suitable for stock portfolio

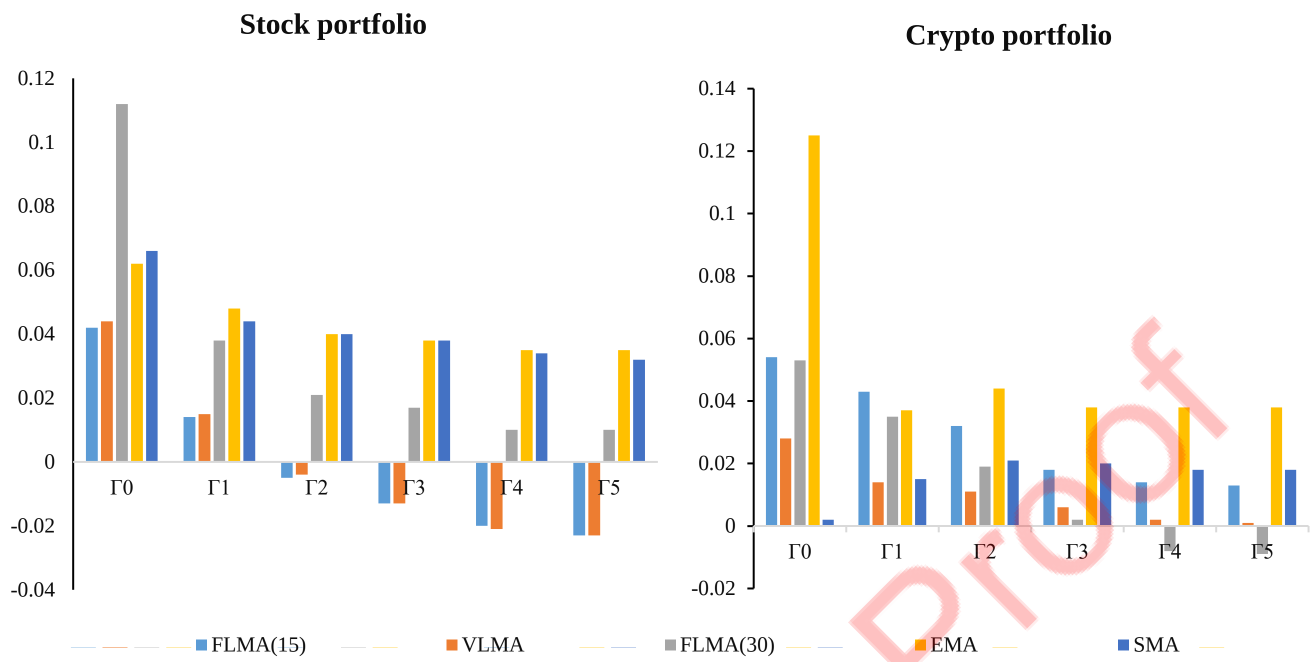

Figure 3. Excess return of each algorithm

Development of a diversified portfolio

As both portfolios studied have similar assets, they are low in diversity, so we combined them for this purpose. The excess return of the newly created portfolio is shown in Figure 4.

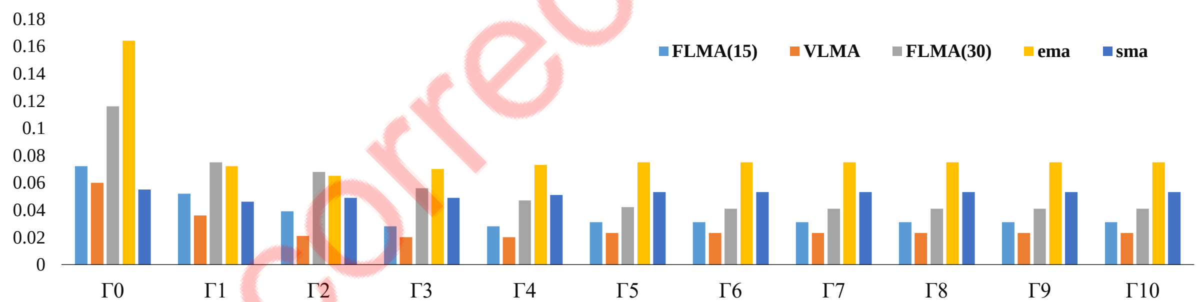

Figure 4. The excess return of the diversified portfolio for each algorithm

The results indicate that the diversified portfolio is a very suitable option for all moving average algorithms, regardless of the degree of uncertainty associated with the model parameters.

5. Discussion and Managerial Insights

This study introduces a groundbreaking approach to algorithmic trading by integrating LSTM-based price predictions with robust portfolio optimization strategies that leverage various moving averages, including SMA, EMA, VLMA, and FLMA. The findings reveal that these strategies consistently outperform the traditional buy-and-hold (B&H) method in both stable and uncertain market conditions. Notably, the SMA

(5,15) strategy for stocks and the FLMA (6,18,0.01) strategy for cryptocurrencies emerged as top performers, delivering the highest returns over short holding periods. These results highlight the effectiveness of short-term moving averages in capturing market trends and optimizing portfolio outcomes. The research further underscores the importance of diversification in portfolio management. By combining stock and cryptocurrency assets within a robust optimization framework, the study demonstrates that diversified portfolios can maintain strong performance across various market scenarios, including periods of heightened uncertainty. The robust portfolio models, especially those based on SMA and EMA, exhibit remarkable resilience, making them invaluable for investors seeking to balance risk and reward in volatile markets. This approach ensures that the portfolio remains a reliable and profitable investment option even under uncertain model parameters. Additionally, this study significantly contributes to algorithmic trading by offering practical insights into the real-world application of LSTM-based predictions. The seamless integration of advanced predictive models with robust portfolio optimization provides a powerful strategy for navigating the complexities of contemporary financial markets. Future research could expand on these findings by exploring a broader range of assets, investigating longer-term investment strategies, and incorporating real-time data to enhance the model's robustness and versatility further. Ultimately, this study presents a compelling argument for adopting diversified, algorithmically optimized portfolios in today's rapidly evolving financial landscape.

5.1. Limitations and Future Works

This study presents certain limitations that open up avenues for future research. Firstly, while economic forces such as macroeconomic trends and geopolitical events can significantly impact data generation and influence financial markets, the current research focused solely on implementing algorithmic trading strategies based on price predictions. Additionally, due to their simplicity, flexibility, and compatibility with other tools, this study exclusively evaluated the performance of moving average-based techniques on an LSTM model, excluding other notable algorithmic trading strategies like MACD, statistical arbitrage, and etc. These alternative strategies could offer additional insights and potentially enhance performance across diverse market conditions. Future research could integrate economic forces to further improve prediction accuracy and portfolio resilience. Moreover, incorporating alternative strategies and economic indicators such as arbitrage trading strategy, mean reversion strategy, weighted average price strategy, and statistical arbitrage strategy into the proposed framework could lead to a more comprehensive and robust portfolio optimization approach.

6. Conclusion

This study presents a new approach to algorithmic trading. In this article, we first investigated the short-term performance of trading algorithms based on moving averages, including EMA, SMA, VLMA, FLMA (15), and FLMA (30) on predicted prices using LSM. Then, we formed a robust portfolio optimization model for each algorithm. As a result, when we invest in stocks, the $SMA(5{,}15)$ strategy will have a better return of 1.119, and when we invest in crypto, $FLMA(6,18,0.01)$ will have the highest return over a 30-day holding period. When evaluating the performance of strategies for all assets, the $FLMA$ $(5,15,0.01)$ strategy with 30 holding times will have the highest GPR with a value of 1.12.

For managing uncertainties, we developed a robust portfolio optimization model for each algorithm. The results indicate that robust portfolio models based on SMA and EMA perform better than other algorithms in both determined and uncertain scenarios. A portfolio optimization model utilizing VLMA and FLMA algorithms will also outperform benchmark strategy (B&H) under certain conditions and also under conditions of low uncertainty. Also, the FLMA (15-day holding), the VLM for stocks, and the FLMA (30-day holding) for crypto portfolios will not be suitable investment options when uncertainty is high. In our investigation, we have integrated the two portfolios to comply with the key principle of diversity. The results indicate that the proposed portfolio is a very suitable option for all moving average algorithms, even when the model parameters are highly uncertain. Therefore, to have a profitable and low-risk investment, it is very suitable to use the new diversified-robust portfolio.

Despite the promising results, this study faces limitations, notably the reliance on short-term moving average strategies, which may restrict the applicability to longer-term investment approaches. Additionally, while the robust portfolio optimization model performs well under certain conditions, its effectiveness in highly volatile or unprecedented market scenarios remains untested.

Future research should explore alternative predictive models, such as reinforcement learning or hybrid approaches that integrate LSTM with other techniques, to improve accuracy and robustness. Expanding the analysis to include a broader range of assets and incorporating real-time data sources like sentiment analysis could enhance the model's adaptability and practical utility in algorithmic trading.

Declaration of generative AI in scientific writing

The authors used the A.I. system [Chat Gpt] in the writing process to improve the readability and language of the manuscript. After, the authors reviewed and edited the content as needed and took full responsibility for the content of the published article.

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