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# The Effect of Size, Value and Idiosyncratic Risk Anomalies on the Relationship between Tail Risk and Stock Excess Returns

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## Abstract

Capital asset pricing models have not considered factors that cause capital market anomalies. The theories of extreme value are one of the arguments for explaining anomalies. Based on the extreme value theory, a tail risk is an adverse event that negatively impacts excess stock returns. Therefore, this study aimed to investigate combining the anomalies of size, value and idiosyncratic risk with tail on stock excess returns. In this study, we have used two criteria of Aggregate Tail Risk and Hybrid Tail Covariance Risk to measure the tail risk. For this purpose, using the systematic removal method, a sample of 136 firms listed on the Tehran Stock Exchange in the period from 2008 to 2019 was selected. The research hypotheses were tested using the Five-Factor Fama and French model (2015). The results suggested that the combination of size and tail risk portfolio and the combination of value and tail risk portfolio have a negative effect on excess return on risk. The results also showed that the combination of idiosyncratic and tail risk portfolios positively and significantly affect stock excess returns. Therefore, by combining these portfolios, investors can gain excess returns in the Iranian capital market. The results generally indicated that tail risk could be added to asset pricing models in addition to the variables of the five-factor Fama and French model.

**Keywords:**
*Size Anomaly, Value Anomaly, Idiosyncratic Risk Anomaly, Tail Risk, Stock Excess Returns.*

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## 1. Introduction

Risk measurement is critical to minimise portfolio risk and assess stability in financial markets. In some cases, such as market collapse, the returns are distributed with a broader tail. Therefore, the criteria used to calculate risk in market stability can not provide helpful information in times of crisis and market collapse. The collapse in the financial markets causes heavy losses in investors' portfolios. Therefore, it is essential to estimate the probability of these events occurring, which cannot be explained and measured by the normal distribution. As a result, understanding the concept of tail risk is a necessity (Massacci, 2017). Technically, tail risk is the portfolio value shifting risk at least three standard deviations from the average, and its occurrence is possible that what is predicted in the normal distribution (Akoundi and Haugh, 2010). Given investors' risk aversion and asymmetric returns distribution, the tail risk becomes important in asset pricing (Long, Jiang and Zhu, 2018). Additionally, after the financial crisis, it has become clear that market returns have a much wider tail than the normal distribution, and tail events occur much more than the normal curve predicts. Tail risk has an asymmetric distribution in the form of a Pareto Distribution (Aboura and Arisoy, 2019). Tail risk can have significant consequences in asset pricing for a number of reasons. For example, Bloom (2009), based on the principles of tail risk, considers uncertainty at the company level due to economic uncertainty, which negatively impacts the company's investment decisions. In asset pricing theory models, it is also specified that non-variable risk from an investor's perspective leads to risk premium demand. Research on asset pricing has suggested several alternative methods for identifying risk factors, a small number of which are now as widespread among academics as market returns (Fama and French, 1993; Carhart, 1997). In addition, recent research has shown that total volatilities, idiosyncratic risk volatilities and market liquidity are also pricing factors (Ang et al., 2009). In research, Gao, Lu and Song (2019) indicated a negative relationship between tail risk and stock returns, similar to the puzzle of idiosyncratic risk volatility in Ang et al. (2009) research. To investigate whether the puzzle of idiosyncratic risk volatility can be explained by tail risk, Long et al. (2019) added idiosyncratic risk volatility to their research model. Their findings show that idiosyncratic risk volatility can explain the negative impact of tail risk pricing in international markets. Bhansali and Davis (2010) found that securing a portfolio against tail risk can increase stock portfolio profitability because a secured portfolio against tail risk will better asset allocation. Finally, Kelly and Jiang (2014) showed that tail risk has a positive and significant relationship with expected returns. Given the above, the primary purpose of this study is to investigate the effect of combining the anomalies of size, value, and idiosyncratic risk with tail risk on the stock excess returns. In addition to expanding the literature on tail risk, this study's results can help form investment portfolios taking into account tail risk. In the following, the theoretical foundations and background of the research are presented, and the research hypotheses are expressed accordingly. Then, after presenting the research findings, the research conclusions and suggestions are expressed.

## 2. Related Literature and Hypothesis Development

Tail risk, defined as the risk of severe events in asset markets, is an essential aspect that investors should consider when making an investment decision; the literature on tail risk and its measurement date back to the early 1960s. Mandelbrot (1963) challenged the common assumption of gaussian return distributions using the power law to describe the unconditional tail distribution of financial returns. Another name for normal distribution is "Gaussian Distribution", and the Power Law is used in Pareto Distributions. The power law of Pareto distribution is essentially a model that indicates the probability of a variable occurring above a certain threshold value (for example, 5% or 10%). The Pareto Distribution is also known as the tail (sequence) function (Tanabe, 2018). Fama

(1963) argues inconsistency with Mandelbrot (1963) that prices in specific markets show sudden and large movements that cannot be explained under the Gaussian Return Distribution model. Sortino and Price (1994) supported downside deviation as a risk criterion, rather than traditional (Gaussian distribution-based) risk criteria such as standard deviation and beta. Sortino Risk criterion never reached the level of acceptance of other criteria such as value at risk, perhaps because it did not consider the full distribution of returns. Agarwal and Naik (2004) found that the left tail was not considered in the context of variance and mean, and therefore, introduced the use of conditional value at risk. Kelly and Jiang (2014) presented the market tail risk criterion based on the common components of individual stocks tail risk and showed considerable predictive power for market returns.

The anomalies indicate market inefficiency or the inadequacy of traditional asset pricing models (Cutler, Poterba and Summers, 1989; Shiller, 2000). In the 1990s, research on anomalies was considered as a research stream. For example, research has been done on size anomalies (Banz, 1981) and value anomalies (Basu, 1983; Lakonishok, Shleifer and Vishny, 1994). Banz (1981) and Reinganum (1981) found that small firms tended to have higher returns than large firms, and this phenomenon could not be explained by the capital assets pricing model (CAPM). There are several possible explanations for the occurrence of this phenomenon. These arguments include the possibility of specific risk factors in small firms compared to large firms (Fama and French, 1993, 1996), Amihud and Mendelson stock liquidity (1986), and the behavioural argument of persistent error in stock valuation (Porta et al., 1997; Lakonishok, Shleifer and Vishny, 1994) can be mentioned. The results Kelly and Jiang (2014) indicate that there is a deep relationship between size and tail risk because smaller firms are more prone to exposure to tail risk shocks, and the most important reason is that their distribution has more kurtosis and skewness (Chen, Hong and Stein, 2001). In addition, Conrad, Dittmar and Ghysels (2013) found a significant negative and stable relationship between risk skewness and future returns. Also, there is a significant positive and stable relationship between risk kurtosis and future returns. Hence, in small firms whose distribution has kurtosis and skewness, it is more likely that higher returns will be provided for the systematic risk of the tail (Aboura and Arisoy, 2019).

Better performance of the stock portfolio of value firms versus the stock portfolio of growth firms is recognised as a value anomaly (Lakonishok, Shleifer and Vishny, 1994). Research results suggest that value portfolios usually have higher returns than growth portfolios (Sharma and Jain, 2020). Fama and French (1996) argue that Risk Premium represents the financial distress risk. In addition, value stocks tend to be less profitable, and investment is less, while growth stocks tend to be more profitable, and investment is more aggressive (Fama and French, 2015). Chen, Hong and Stein (2001) found that the ratio of book value to the market value of a smaller (larger) stock is associated with a negative (positive) kurtosis. The asymmetric differences between the distribution of the stock returns of growth and value firms indicate that their returns are at risk of aggregate tail risk. Economic theories also suggest that value premium can be associated with tail risk. The results of previous research suggest that value premium is related to the economic conditions of recession and prosperity (Aboura and Arisoy, 2019). Considering that crashes occur in times of market stress (Daniel and Moskowitz, 2016), and also, considering that value premium is a function of economic conditions, it can be said that value firms are more exposed to the negative effects of tail risk compared the growth firms.

Merton (1987) indicates that if there is an anomaly in a market where investors have limited access to information, stocks with high idiosyncratic volatility provide high expected returns because Levy (1978) theoretically shows that if investors do not have a lot of assets in their portfolio, idiosyncratic risk affects the equilibrium price of assets. Malkiel and Xu (1997) prove that stocks with higher idiosyncratic risk volatility have higher average returns. However, they do not provide a specific level for their idiosyncratic risk volatility premium. Also, the results of Malkiel and Xu (2002) suggest that there is a positive and significant relationship between idiosyncratic risk and cross-sectional expected returns at the company level. Spiegel and Wang (2005) focused on predicting outside the sample of idiosyncratic risk volatility and liquidity and found that expected stock returns increased with idiosyncratic risk levels and decreased stock liquidity. Their findings show that liquidity and idiosyncratic risk play a major role in determining stock returns, but the effect of idiosyncratic risk is much stronger. Some behavioural models, such as Barberis and Huang (2001), also predict higher systematic risk volatility stocks with higher expected returns. Various arguments about the negative relationship between idiosyncratic risk volatility and returns include short term return reversal (Huang et al., 2009), earnings surprises (Jiang, Xu and Yao, 2009), non-systematic skewness (Boyer, Mitton, and Vorkink 2010), Average Variance Beta (Chen and Petkova, 2012), incomplete information (Berrada and Hugonnier, 2013), and prospect theory (Bhootra and Hur, 2015). The tail risk may be associated with idiosyncratic risk through uncertainty. This relationship is due to higher volatilities in stock returns. Also, if the expected investment opportunities are stable, companies will delay their investment by increasing uncertainty and market volatility (Kim and Kung, 2017). Baltussen, van Bekkum and van der Grient (2018) used unstable volatilities as stock uncertainty. Their results showed that the uncertainty caused by volatilities could lead to the explanation of cross-sectional stock returns. On the other hand, volatilities are closely related to the fourth-order moment of the return distribution. Therefore, distribution tails affect idiosyncratic risk volatility with increasing uncertainty and unstable volatilities. Hence, idiosyncratic risk volatility of the stock is associated with tail risk. Kelly and Jiang (2014) found that tail risk and idiosyncratic risk are different components of stock returns. But they did not examine the relationship between tail risk and idiosyncratic risk. The Aboura and Arisoy (2019) results showed that the combination of idiosyncratic risk anomalies and tail risk affects the stock excess returns. Some foreign research related to the research topic is examined in the following.

Ogbonna and Olubusoye (2021) showed that the specific risk of each country has the most positive effect on returns. Also, tail risk on bad days increases the short-term negative return, and on good days its effects disappear completely.

Sun, Wang and Zhu (2021) showed that if stocks with small capital are removed from the Chinese market, there is a significant negative relationship between tail risk and expected returns. Also, their results showed that psychological and behavioural biases, investors' lack of reaction to bad news, relative preference for tail risk and high emotions are the reasons for the negative relationship between tail risks and expected stock returns.

Aboura and Arisoy (2019) investigate the effect of tail risk on portfolios arranged based on anomalies of size, value and idiosyncratic risk volatility. The study was conducted in the United States from 1963 to 2013. The results indicate that portfolios that include small and value stocks have a negative and significant beta for their tail risk. Also, portfolios that include stocks with high systematic risk volatility have a negative and significant beta for their tail risk. Further, their cross-sectional analysis of individual stocks shows that tail risk helps explain pricing anomalies, especially idiosyncratic risk volatility.

Long, Jiang and Zhu (2018) showed a significant negative relationship between idiosyncratic tail risk and expected returns in the Chinese stock market after controlling for other risk criteria such as size, momentum and liquidity.

Aboura and Chevallier (2018) conducted a study entitled “tail risk and the relationships of returns”; they brought a new perspective on time-varying leverage and feedback effects in US stock markets. Their experimental findings showed that the lever effect is symmetric while the feedback is asymmetric. Dynamic leverage also has the greatest impact on the advancement of stock markets, and both the leverage effect and the feedback effect increase with increasing unstable volatilities.

Bollerslev, Todorov and Xu (2015) showed that the explanatory power of the regressions used to predict returns increases with the addition of tail risk components as an independent predictor variable.

### 2.1. Research hypotheses

Based on the theoretical foundations and research background, the research hypotheses are formulated as follows:

**$H_1$:** Using the combination of size portfolio and tail risk, excess return on risk can be obtained.

**$H_2$:** Using the combination of value portfolio and tail risk, excess return on risk can be obtained.

**$H_3$:** By combining a portfolio of idiosyncratic and tail risks, excess returns on risk can be obtained.

## 3. Research Methodology

In order to test the research hypotheses, data were collected every month, and multivariate regression models were used to analyse the data. The sample of this research is the companies listed on the Tehran Stock Exchange. The period is from 2008 to 2019. The systematic elimination method was used for sampling. Therefore, companies with the following conditions have been selected as samples:

1. In terms of increasing comparability, their financial period should end in March.
2. The company is a stock exchange member from beginning to end of the research.
3. The required information is available about such companies.
4. Companies should not be part of banks and financial institutions.
5. The company’s financial year or activity should not be changed during the research period.
6. The stock symbol of the companies listed on the stock exchange should not stop for more than 3 months.

By applying the above, 136 companies were selected as a research sample. The required data were extracted from financial statements and information software and analysed using Eviews software.

### 3.1. Research model and research variables

In order to analyse and test the research hypotheses, the models presented in the research of Aboura and Arisoy (2019) have been used. In their research, the five-factor Fama and French model (2015) and two alternative criteria of tail risk, Aggregate Tail Risk Index of Kelly and Jiang (2014) and Hybrid Tail Covariance Risk of Bali, Cakici and Whitelaw (2014), were used. The assets tested in this study include portfolios classified according to the combination of tail risk and anomalies of size, value, profitability and investment. GRS (Gibbons, Ross, and Shanken, 1989) test is used to evaluate the validity of models and their ability to explain the excess returns of portfolios. The adjusted coefficient of determination of GRS reports how much of the excess returns of portfolios is explained by the intercept of the model. In other words, the lower the coefficient for the model under test, the better the model's fit. That is, independent variables better explain the effects of the dependent variable. In order to test the research hypotheses, the time regression series specified in Equation (1) is used:

$$R_{Pt} - R_{Ft} = \beta_1 + \beta_2 MKT_t + \beta_3 SMB_t + \beta_4 HML_t + \beta_5 RMW_t + \beta_6 CMA_t + \varepsilon_t \quad (1)$$

In model number (1), $\mathrm{R_{Pt}} - \mathrm{R_{Ft}}$, represents the excess return on the portfolio risk of "P" in a month "t", $\mathrm{R_{Pt}}$ represents the average return of portfolio "P" in a month "t", $\mathrm{R_{Ft}}$ represents the risk-free rate of return in a month "t". $MKT_t$ represents the market factor, $SMB_t$ the size factor, $HML_t$represents the value factor, $RMW_t$ represents the profitability factor, and $CMA_t$ represents the investment factor.

Excess return on risk ($R_{Pt} - R_{Ft}$): the test of the five-factor model performed at the portfolio level. In other words, excess returns on portfolios are the research's dependent variable. In order to form a portfolio of size and tail risk, at the end of each research year, after sorting the entire sample stock based on the company's stock market value from small to large firms, it is classified into two groups, small and large, using the median. Then, regardless of this classification, the total sample stock of the research is divided into three categories each year based on the tail risk variable, so that the beginning 30% is called a small portfolio, the middle 40% is called a medium portfolio, and the final 30% is called a large portfolio. In the end, a portfolio of size and tail risk is formed, which is used to test the first hypothesis. In order to form a value portfolio and tail risk, at the end of each year, after sorting the total sample stocks based on the ratio of book value to market value into three groups of 30% at the beginning, 40% at the middle and 30% final, in the continuation and independent of this classification, the sample is classified based on the tail risk variable in each year into o three groups: 30% beginning, 40% middle, and 30% final. Combining them will form a value portfolio and tail risk, which is used to test the second hypothesis. In order to form an idiosyncratic risk portfolio and tail risk, at the end of each research year, after sorting the total sample stocks according to the idiosyncratic risk, they are divided into three groups, including 30% beginning, 40% middle, and 30% final. In the continuation and independent from this classification, the total sample is classified into three groups at each year and based on the tail risk variable. So that the first 30% is called a small portfolio, the middle 40% is called a medium portfolio and the last 30% is called a large portfolio. Combining them will form an idiosyncratic risk portfolio and tail risk used to test the third hypothesis. It can also be explained that the way the portfolios are formed is similar for both variables used to calculate the tail risk. In the following, we will describe how to calculate the tail risk. Long et al. (2019) and Kelly and Jiang (2014) research were used to measure the first criterion of Aggregate Tail Risk. First, the monthly tail risk for each research sample stock is estimated in time series. For moth "t", all daily stock returns on the days traded during that month are collected as a sample, and its tail index is estimated using the Hill (1975) developed method as follows:

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{K=1}^{K_t} ln \frac{R_{k,t}}{u_t} \tag{2}$$

So that $R_{k,t}$ is the k^th^ daily return, which is below the threshold of the limit value $u_t$ during month "t" and $K_t$ is the total number of these cases during the month "t". Following Kelly and Jiang (2014), $u_t$ is defined at a significance level of 10% and 5% for each period. In the following, the tail risk of the company "i" (determined by TR) in month "t" is estimated based on the following model for each share and based on time series data:

$$R_{i,t} = \mu_i + TR_{i,t}\lambda_t^{Hill} + \varepsilon_{i,t} \tag{3}$$

Where, $R_{i,t}$, is the monthly return of the company "i" in the month "t" and $\lambda_t^{Hill}$ is the tail index that is obtained from Equation (2). In order to estimate $TR_{i,t}$ for share "i" in a month "t", the rolling regression with a period of 60 months has been used. This method was used to estimate each of the research companies. Stock with high-value aggregate tail risk (TR) is more sensitive to tail risk.

In their study, Aboura and Arisoy (2019) have measured the second criterion of tail risk, i.e. hybrid tail covariance risk (HTCR). They have used the method of Bali, Cakici and Whitelaw

(2014). Hybrid tail covariance risk (HTCR) measures stock returns and market returns subject to negative stock returns at the stock level. Using a significance level of 10% or 5% for "k" in the "i" stock distribution, the hybrid tail covariance risk (HTCR) is defined as follows:

$$HTCR = \sum_{R_i<k_i}(R_i - k_i)(R_m - k_m) \tag{4}$$

Where the $R_M$ represents the market portfolio and $R_i$ is the return portfolio. Following the research of Bali, Cakici and Whitelaw (2014), the daily returns of the last six months have been used to calculate the tail risk of each stock.

$MKT_t$ represents the market factor that is obtained from the difference between market returns and risk-free monthly returns ($R_M - R_F$). In this study, the risk-free monthly return is obtained based on the data of the Iranian Central Bank and by dividing the interest rate of one-year bank deposits by 12.

SMB represents the size factor obtained from the difference between the average return on small firms' stock portfolios and the stock portfolio of large firms. Smaller components, namely calculate the calculation of the total SMB $SMB_{B/M}$, $SMB_{OP}$, and $SMB_{INV}$ that is obtained through the following equations:

$$SMB_{B/M} = {(SH + SN + SL)}/{3} - {(BH + BN + BL)}/{3} \tag{5}$$

$$SMB_{OP} = {(SR + SN + SW)}/{3} - {(BR + BN + BW)}/{3} \tag{6}$$

$$SMB_{INV} = {(SA + SN + SC)}/{3} - {(BA + BN + BC)}/{3} \tag{7}$$

$$SMB = {(SMB_{B/M} + SMB_{OP} + SMB_{INV})}/{3} \tag{8}$$

HML represents the book value factor to market value, which is obtained from the difference between the return of the stock portfolio with the ratio of book value to high market value and the portfolio with the ratio of book value to low market value in a month "t". The following relation performs value factor (HML) calculations:

$$HML = {(SH + BH)}/{2} - {(SL + BL)}/{2} \tag{9}$$

RMW is the profitability factor obtained from the difference between the average return of a portfolio with high operating profitability and a portfolio with low operating profitability and the combination of the size factor. Profitability factor (RMW) calculations are performed through the following relation:

$$RMW = {(SR + BR)}/{2} - {(SW + BW)}/{2} \tag{10}$$

CMA represents the investment factor derived from the difference between the average return on a high-investment portfolio and a low investment portfolio and a combination of the size factor. Investment factor (CMA) calculations are performed using the following equation:

$$CMA = {(SC + BC)}/{2} - {(SA + BA)}/{2} \tag{11}$$

Idiosyncratic risk (IVOL): The market model calculates idiosyncratic risk in this study. For this purpose, first, the width of the origin and the coefficient of the market model in each month are calculated separately for each stock.

$$R_j = \propto + \beta_t R_{mt} + \varepsilon_t \tag{12}$$

$R_j$ represents the daily return of each company and $R_{mt}$ represents the daily return of the market. Then, the market model residues are calculated for the same trading days. Finally, the idiosyncratic risk is multiplied by the standard deviation of the daily residues squared by the number of monthly trading days.

$$IVOL = STE * \sqrt{TD} \tag{13}$$

In the above relation, STE indicates the standard deviation of the market model residues per month, and TD indicates the number of monthly trading days. Finally, the research hypotheses were tested using the five-factor Fama and French (2015) model.

## 4. Research Findings

Table 1 shows the results of descriptive statistics of research variables.

**Table1.** Descriptive statistics

| Variables | Mean | Median | Standard deviation | Maximum | Minimum |
|---|---|---|---|---|---|
| Rm – Rf | 0.015 | 0.003 | 0.067 | 0.248 | -0.101 |
| SMB | 0.002 | -0.002 | 0.033 | 0.092 | -0.071 |
| HML | -0.039 | -0.037 | 0.054 | 0.166 | -0.222 |
| RMW | 0.001 | 0.002 | 0.046 | 0.118 | -0.108 |
| CMA | 0.006 | 0.004 | 0.046 | 0.161 | -0.104 |

The table above shows that the mean and standard deviation are 0.015 and 0.067 for the market factor, 0.002 and 0.033 for the size factor, and -0.040 and 0.054 for the growth factor, respectively. It is 0.001 and 0.046 for the profitability factor and 0.006 and 0.046 for the investment factor. These cases show that due to the negative mean of the growth factor, its monthly return is on average 0.040% lower than the risk-free return. On the other hand, the size factor has the lowest standard deviation due to higher diversification, and the market factor, which has the highest standard deviation, has higher volatilities.

### 4.1. Testing research hypotheses

Examination of the significance of research variables shows that the research variables are at the level of durable, and since the results of the autocorrelation test indicate the existence of autocorrelation, to solve this problem, first-ordered autoregressive has been added to the research models.

The first hypothesis of the research stated that the combination of size portfolio and tail risk affects excess return on risk. The results of testing the first hypothesis are reported in Table 2.

The intercept coefficient of the five-factor Fama and French (2015) model for tail risk (TR (5)), (TR (10)), and (HTCR (5)) is -0.005 and is -0.006 for tail risk (HTCR (10)) that is negative and significant in the domain under study. Therefore, considering the significance of the coefficients, it can be concluded that the combination of size portfolio and tail risk leads to an excess return on risk. As a result, the first hypothesis of the research is accepted.

The second hypothesis of the research stated that the combination of value portfolio and tail risk affects excess return on risk. The results of testing the second hypothesis are reported in Table 3.

**Table 2.** Test results of the first hypothesis

$$R_{Pt} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \beta_2(SMB_t) + \beta_3(HML_t) + \beta_4(RMW_t) + \beta_4(CMA_t) + \varepsilon_t$$

| | Tail Risk (TR(5)) | | Tail Risk (TR(10)) | | Tail Risk (HTCR(5)) | | Tail Risk (HTCR(10)) | |
|---|---|---|---|---|---|---|---|---|
| Variable | Coefficients | t-statistic | Coefficients | t-statistic | Coefficients | t-statistic | Coefficients | t-statistic |
| C | -0.005 | -2.486* | -0.005 | -2.517* | -0.005 | -2.603* | -0.006 | -2.606* |
| $R_m - R_f$ | 1.022 | 24.103* | 1.014 | 25.833* | 1.021 | 22.968* | 1.017 | 20.901* |
| SMB | 0.764 | 5.778* | 0.759 | 5.961* | 0.750 | 5.587* | 0.782 | 5.531* |
| HML | -0.183 | -2.472* | -0.194 | -2.681* | -0.184 | -2.366* | -0.166 | -1.910* |
| RMW | -0.056 | -0.869 | -0.055 | -0.850 | -0.046 | -0.702 | -0.051 | -0.771 |
| CMA | 0.501 | 6.604* | 0.501 | 7.041* | 0.497 | 6.405* | 0.495 | 6.216* |
| AR(1) | -0.040 | -0.605 | -0.039 | -0.601 | -0.082 | -1.335 | -0.054 | -0.890 |
| F-Statistic | | 167.378* | | 167.981* | | 158.357* | | 150.659* |
| Adjusted $R^2$ | 0.607 | | 0.608 | | 0.594 | | 0.582 | |
| Durbin Watson | 1.996 | | 1.998 | | 1.994 | | 1.993 | |

*= significance is at the level of 5%**.**

**Table 3**. Test results of the second hypothesis

$$R_{Pt} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \beta_2(SMB_t) + \beta_3(HML_t) + \beta_4(RMW_t) + \beta_4(CMA_t) + \varepsilon_t$$

| | Tail Risk (TR(5)) | | Tail Risk (TR(10)) | | Tail Risk (HTCR(5)) | | Tail Risk (HTCR(10)) | |
|---|---|---|---|---|---|---|---|---|
| Variable | Coefficients | t-statistic | Coefficients | t-statistic | Coefficients | t-statistic | Coefficients | t-statistic |
| C | -0.007 | -2.070* | -0.008 | -1.993* | -0.007 | -1.962* | -0.007 | -2.628* |
| $R_m - R_f$ | 0.999 | 23.455* | 0.992 | 21.955* | 1.010 | 20.704* | 1.011 | 19.298* |
| SMB | 0.785 | 6.858* | 0.820 | 6.870* | 0.793 | 6.441* | 0.811 | 6.276* |
| HML | -0.155 | -1.953* | -0.144 | -1.623 | -0.154 | -1.604 | -0.147 | -1.427 |
| RMW | -0.076 | -1.292 | -0.089 | -1.499 | -0.065 | -1.099 | -0.062 | -1.025 |
| CMA | 0.482 | 7.698* | 0.485 | 7.323* | 0.478 | 7.118* | 0.486 | 7.149* |
| AR(1) | 0.044 | 0.932 | 0.066 | 1.114 | -0.012 | -0.225 | -0.006 | -0.120 |
| F-Statistic | | 195.460* | | 176.483* | | 171.819* | | 162.658* |
| Adjusted $R^2$ | 0.546 | | 0.520 | | 0.514 | | 0.500 | |
| Durbin Watson | 2.003 | | 2.006 | | 1.998 | | 1.998 | |

*= significance is at the level of 5%**.**

The intercept coefficient of the origin of the five-factor Fama and French (2015) model for tail risk (TR(5)), (HTCR(10)), and (HTCR(5)) is equal to -0.007 and for tail risk (TR(10)) is equal to -0.008 that is negative and significant in the domain under study.

Therefore, considering the significance of the coefficients, it can be concluded that the combination of value portfolio and tail risk leads to an excess return on risk. As a result, the second hypothesis of the research is not rejected.

The third hypothesis of the research is that the combination of idiosyncratic risk portfolio and tail risk affects excess return on risk. The test results of the third hypothesis are presented in Table 4.

The findings in Table 4 show the intercept coefficient of the origin of the five-factor Fama and French (2015) model for tail risk (TR (5)), (TR (10)), and is 0.009 and for tail risk (HTCR(10)) is 0.010, which is positive and significant in the domain under study. Therefore, due to the significance of the coefficients, it is possible to obtain an excess return on risk through the combination of idiosyncratic risk portfolio and tail risk. The findings indicate the acceptance of the third hypothesis. On the other hand, the F statistic shows that the model is significant and is not a

false regression. In addition, the value of the coefficient of determination indicates the optimal description of the dependent variable by independent variables.
The GRS test is used to assess the validity of models and their ability to explain the excess returns of portfolios. The results of the GRS test of research hypotheses are reported in Table 5.

**Table 4.** Test results of the third hypothesis

$$R_{Pt} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \beta_2(SMB_t) + \beta_3(HML_t) + \beta_4(RMW_t) + \beta_4(CMA_t) + \varepsilon_t$$

| | Tail Risk (TR(5)) | | Tail Risk (TR(10)) | | Tail Risk (HTCR(5)) | | Tail Risk (HTCR(10)) | |
|---|---|---|---|---|---|---|---|---|
| **Variable** | **Coefficients** | **t-statistic** | **Coefficients** | **t-statistic** | **Coefficients** | **t-statistic** | **Coefficients** | **t-statistic** |
| C | 0.009 | 2.250* | 0.009 | 2.103* | 0.009 | 2.388* | 0.010 | 2.342* |
| $R_{m-}R_f$ | 0.949 | 20.574* | 0.965 | 21.256* | 0.961 | 20.650* | 0.950 | 19.165* |
| SMB | 0.848 | 6.995* | 0.828 | 6.685* | 0.809 | 6.487* | 0.843 | 6.467* |
| HML | -0.126 | -1.602 | -0.134 | -1.699 | -0.135 | -1.871 | -0.121 | -1.510 |
| RMW | -0.104 | -1.659 | -0.085 | -1.395 | -0.080 | -1.250 | -0.077 | -1.218 |
| CMA | 0.495 | 6.885* | 0.501 | 7.230* | 0.499 | 6.735* | 0.493 | 6.640* |
| AR(1) | 0.173 | 3.158* | 0.160 | 2.887* | 0.144 | 2.743* | 0.164 | 3.042* |
| F-Statistic | | 171.948* | | 174.302* | | 162.842* | | 156.455* |
| Adjusted $R^2$ | 0.514 | | 0.517 | | 0.500 | | 0.490 | |
| Durbin Watson | 2.037 | | 2.038 | | 2.040 | | 2.050 | |

*= significance is at the level of 5%.

**Table 5.** GRS test results

| | The First Hypothesis | | The Second Hypothesis | | The Third Hypothesis | |
|---|---|---|---|---|---|---|
| | **GRS Coefficient** | **Adjusted $R^2$** | **GRS Coefficient** | **Adjusted $R^2$** | **GRS Coefficient** | **Adjusted $R^2$** |
| Tail Risk Model (TR(5)) | 0.006* | 0.637 | 0.006* | 0.600 | 0.007* | 0.564 |
| Tail Risk Model (TR(10)) | 0.006* | 0.632 | 0.007* | 0.584 | 0.006* | 0.571 |
| Tail Risk Model (HTCR(5)) | 0.006* | 0.629 | 0.021* | 0.528 | 0.007* | 0.562 |
| Tail Risk Model (HTCR(10)) | 0.007* | 0.620 | 0.021* | 0.555 | 0.008* | 0.557 |

*= significance is at the level of 5%.

The results of the GRS test of the first hypothesis show that the intercept in the portfolios formed with the combination of size and tail risk is significant. Hence, it can be expected that the combination of size variable and tail risk will affect the excess return on risk. Adjusted coefficients of determination for different models in Table 5 varies from 0.620 to 0.637. This indicates that despite the closeness of the explanatory power of the models, the validity of the tail risk model (HTCR (10)) is slightly higher than the others.

The results of the GRS test of the second hypothesis show that the intercept in the portfolios formed with the combination of value and tail risk is significant. Adjusted coefficients of determination for different models of tail risk is between 0.528 and 0.600. These results indicate that the tail risk model (HTCR (5)) has more explanatory power than other models.

The results of the GRS test of the third hypothesis show that the intercept is significant in portfolios that consist of a combination of idiosyncratic risk and tail risk. Therefore, it shows that the combination of idiosyncratic risk variable and tail risk affects excess returns on risk. Table 5 adjusted coefficients of determination for different models vary from 0.557 to 0.571. This indicates that although the explanatory power of the models is close to each other, the validity of the tail risk model (HTCR (10)) is slightly higher than other models.

## 5. Conclusion and Discussion

Accurate identification and risk assessment in financial markets can lead to favourable capital allocation and efficiency. On the other hand, increasing volatilities in financial markets and economic crises lead to adverse events and, consequently, wider tails than the normal distribution occur. Among the anomalies affecting the effects of tail risk on the asset, return is size, value and idiosyncratic risk. Therefore, in market collapse and crash situations, tail risk assessment is critical considering the anomalies of size, value and idiosyncratic risk to obtain excess returns. Hence, in this study, the effect of size abnormality, value abnormality, and idiosyncratic risk on the relationship between tail risk and stock excess returns was investigated, and for this purpose, three hypotheses were formulated. The first hypothesis stated whether or not an excess return on risk can be obtained by combining the size portfolio and tail risk in the Tehran Stock Exchange. Accordingly, the first hypothesis of the research is not rejected. The results of the first hypothesis test are consistent with the research of Aboura and Arisoy (2019). Findings from testing the second hypothesis of the research showed that the combination of value portfolio and tail risk leads to an excess return on risk. Accordingly, the second hypothesis of the research is not rejected. The results of the second hypothesis test are consistent with the research of Aboura and Arisoy (2019). The results of testing the third hypothesis of the research showed that the combination of idiosyncratic risk portfolio and tail risk in the Tehran Stock Exchange could provide an excess return on risk. Accordingly, the third hypothesis of the research is confirmed. The results of the third hypothesis test are consistent with the research of Aboura and Arisoy (2019).

Based on the results of this study, considering that the combination of size and tail risk portfolio and the combination of value and tail risk portfolio and the combination of idiosyncratic risk and tail risk portfolio lead to stock excess returns, investors are advised to combine portfolios in their investments. In other words, investors can achieve higher returns by choosing small and value firms that have high tail risk and a combination of firms with high idiosyncratic volatility and tail risk. Also, market analysts and managers of investment companies are advised to consider the effects of Pareto Distribution in their investment decisions that form tail risk.

Researchers are advised to use other criteria for measuring tail risk in future research. Researchers are also advised to examine the combination of tail risk and other abnormalities.

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## References

1. Aboura, S. and Arisoy, Y.E. (2019). Can Tail Risk Explain Size, Book-to-Market, Momentum, and Idiosyncratic Volatility Anomalies?. *Journal of Business Finance & Accounting*, 46(9-10), pp. 1263-1298. https://doi.org/10.1111/jbfa.12403.
2. Aboura, S. and Chevallier, J. (2018). Tail Risk and the ReturnVolatility Relation. *Research in International Business and Finance*, 46(2), pp. 16-29. https://doi.org/10.1016/j.ribaf.2016.07.036.
3. Agarwal, V. and Naik. N.Y. (2004). Risks and Portfolio Decisions Involving Hedge Funds. *Review of Financial Studies*, 17(1), pp. 63-98.
4. Akoundi, K. and Haugh, J. (2010). Tail risk hedging: A roadmap for asset owners. Deutsche Bank Pension Strategies and Solutions.
5. Amihud, Y. and Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2), pp. 223-249. https://doi.org/10.1016/0304-405X(86)90065-6.
6. Ang, A., Hodrick, R., Xing, Y., Zhang, X. (2009). High idiosyncratic volatility and low

returns: International and further US evidence. *Journal of Financial Economics*, 91(1), pp. 1-23.
7. Bali T.G., Cakici, N. and Whitelaw. R.F. (2014). Hybrid Tail Risk and Expected Stock Returns: When Does the Tail wag the Dog?. *Review of Asset Pricing Studies*, 4(2), pp. 206-246. https://doi.org/10.1093/rapstu/rau006.
8. Baltussen, G., van Bekkum, S. and van der Grient, B. (2018). Unknown unknowns: Uncertainty about Risk and Stock Returns, *Journal of Financial and Quantitative Analysis*, 53(4), pp. 1615-1651.
9. Banz, R.W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9(1), pp. 3–18. https://doi.org/10.1016/0304-405X(81)90018-0.
10. Barberis, N. and Huang, M. (2001). Mental Accounting, loss Aversion, and Individual Stock Returns. *Journal of Finance*, 56(4), pp. 1247–1292. https://doi.org/10.1111/0022-1082.00367.
11. Basu, S. (1983). The relationship between earnings yield, market value and return for NYSE common stocks: Further evidence. *Journal of Financial Economics*, 12(1), pp. 129–156. https://doi.org/10.1016/0304-405X(83)90031-4.
12. Berrada, T., Hugonnier, J. (2013). Incomplete information, idiosyncratic volatility and stock returns. *Journal of Banking and Finance*, 37(2), pp. 448-462. https://doi.org/10.1016/j.jbankfin.2012.09.004.
13. Bhootra, A. and Hur, J. (2015). High idiosyncratic volatility and low returns: A prospect theory explanation. *Financial Management*, 44(2), pp. 295-322. https://doi.org/10.1111/fima.12057.
14. Bloom, N. (2009). The impact of uncertainty shocks. Econometrica. *journal of the econometric society*, 77(3), pp. 623-685. https://doi.org/10.3982/ECTA6248.
15. Bollerslev, T., Todorov, V. and Xu, L. (2015). Tail risk premia and return predictability. *The Journal of Financial Economics*, 118(1), pp. 113-134.
16. Boyer, B., Mitton, T. and Vorkink, K. (2010). Expected idiosyncratic skewness. *The Review of Financial Studies*, 23(1), pp. 169-202. https://doi.org/10.1093/rfs/hhp041.
17. *Bhansali, V. and Davis, J. M. (2010). Offensive Risk Management: Can Tail Risk Hedging Be Profitable? Corporate Governance & Finance eJournal.* https://doi.org/10.2139/ssrn.1573760
18. Carhart, M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, 52(1), pp. 57- 82.
19. Chen, J., H., Hong, and Stein. J. C. (2001). Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices. *The Journal of Financial Economics*, 61(3), pp. 345-381. https://doi.org/10.1016/S0304-405X(01)00066-6.
20. Chen, Z. and Petkova, R. (2012). Does idiosyncratic volatility proxy for risk exposure?. *The Review of Financial Studies*, 25(9), pp. 2745-2787. https://doi.org/10.1093/rfs/hhs084.
21. Conrad, J, Dittmar, R.F, and Ghysels, E. (2013). Ex-ante skewness and expected stock returns. *Journal of Finance*, 68(1), pp. 85-124. https://doi.org/10.1111/j.1540-6261.2012.01795.x.
22. Cutler, D.M., Poterba, J.M. and Summers, L.H. (1989). What moves stock prices?. *The Journal of Portfolio Management*, 15(3), pp. 4-12.
23. Daniel, K. and Moskowitz, T.J. (2016). Momentum crashes. *The Journal of Financial Economics*, 122(2), pp. 221-247. https://doi.org/10.1016/j.jfineco.2015.12.002.
24. Fama, E. F. (1963). Mandelbrot and the Stable Paretian Hypothesis. *The Journal of Business*,

36(4), pp. 420–429.
25. Fama, E. F. and French. K. R. (2015). A five-factor asset pricing model. *The Journal of Finance*, 116(1), pp.1-22. https://doi.org/10.1016/j.jfineco.2014.10.010.
26. Fama, E. F. and French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The Journal of Finance*, 51(1), pp. 55-84. https://doi.org/10.1111/j.1540-6261.1996.tb05202.x.
27. Fama, E. F. and French. K. R. (1993). Common risk factors in the returns on stocks and bonds. *The Journal of Financial Economics*, 33(1), pp. 3-56. https://doi.org/10.1016/0304-405X(93)90023-5.
28. Gao, G. P., Lu, X., Song, Z. (2019). Tail Risk Concerns Everywhere. *Management Science*, 65(7), pp. 3111-3130. https://doi.org/10.1287/mnsc.2017.2949.
29. Gibbons, M. R., Ross, S. A. and Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, 57(5), pp. 1121-1152.
30. Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. *The annals of statistics*, 3(5), pp. 1163-1174.
31. Huang, W., Liu, Q., Rhee, S. G. and Zhang, L. (2009). Return reversals, idiosyncratic risk, and expected returns. *The Review of Financial Studies*, 23(1), pp. 147-168. https://doi.org/10.1093/rfs/hhp015.
32. Jiang, G.J., Xu, D. and Yao, T. (2009). The information content of idiosyncratic volatility. *Journal of Financial and Quantitative Analysis*, 44(1), pp. 1-28. https://doi.org/10.1017/S0022109009090073.
33. Kelly, B. and Jiang, H. (2014). Tail risk and asset prices. *The Review of Financial Studies*, 27(10), pp. 2841-2871. https://doi.org/10.1093/rfs/hhu039.
34. Kim, H. and Kung, H. (2017). The asset redeploys ability channel: How uncertainty affects corporate investment. *The Review of Financial Studies*, 30(1), pp. 245-280. https://doi.org/10.1093/rfs/hhv076.
35. Lakonishok, J., Shleifer, A. and Vishny, R. W. (1994). Contrarian investment, extrapolation, and risk. *The Journal of Finance*, 49(5), pp.1541–1578. https://doi.org/10.1111/j.1540-6261.1994.tb04772.x.
36. Levy, H. (1978). Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio. *The American Economic Review*, 68(4), pp. 643-658.
37. Long, H., Jiang, Y. and Zhu, Y. (2018). Idiosyncratic tail risk and expected stock returns: Evidence from the Chinese stock markets. *Finance Research Letters*, 24(16), pp. 129-136. https://doi.org/10.1016/j.frl.2017.07.009.
38. Long, H., Zhu, Y., Chen, L. and Jiang, Y. (2019). Tail risk and expected stock returns around the world. *Pacific-Basin Finance Journal*, 56(9), pp. 162-178. https://doi.org/10.1016/j.pacfin.2019.06.001.
39. Malkiel, B. G. and Xu, Y. (1997). Risk and return revisited. *Journal of Portfolio Management*, 23(3), pp. 9-14. https://doi.org/10.3905/jpm.1997.409608.
40. Malkiel, B. G. and Xu, Y. (2002). Idiosyncratic Risk and Security Returns. Working Paper, the University of Texas at Dallas. The USA.
41. Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business*, 36(4), pp. 394-419.
42. Massacci, D. (2017). Tail Risk Dynamics in Stock Returns: Links to the Macroeconomy and Global Markets Connectedness. *Management Science*, 63(9), pp. 3072–3089. https://doi.org/10.1287/mnsc.2016.2488.
43. Merton, R.C. (1987). Presidential address: a simple model of capital market equilibrium with

incomplete information. *Journal of Finance*, 42(3), pp. 483–510. https://doi.org/10.1111/j.1540-6261.1987.tb04565.x.
44. Ogbonna, A. E. and Olubusoye, O. E. (2021). Tail Risks and Stock Return Predictability: Evidence From Asia-Pacific. *Asian Economics Letters*, 2(3), pp. 1-6 https://doi.org/10.46557/001c.24417.
45. Porta, R. L., Lakonishok, J., Shleifer, A. and Vishny, R. (1997). Good news for value stocks: Further evidence on market efficiency. *The Journal of Finance*, 52(2), pp. 859-874. https://doi.org/10.1111/j.1540-6261.1997.tb04825.x.
46. Reinganum, R. (1981). Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values. *Journal of Financial Economics*, 9(1), pp. 19-46. https://doi.org/10.1016/0304-405X(81)90019-2.
47. Sharma, M. and Jain, A. (2020). Role of size and risk effects in value anomaly: Evidence from the Indian stock market. Cogent Economics & Finance, 8(1), A. 1838694. https://doi.org/10.1080/23322039.2020.1838694.
48. Shiller, R. J. (2000). Measuring bubble expectations and investor confidence. *The Journal of Psychology and Financial Markets*, 1(1), pp. 49–60. https://doi.org/10.1207/S15327760JPFM0101_05.
49. Sortino, F. A. and Price, L. N. (1994) Performance measurement in a downside risk framework. Journal of Investing, 3(3), pp. 59-65. https://doi.org/10.3905/joi.3.3.59.
50. Spiegel, M., Wang. X. (2005). Cross-Sectional Variation in Stock Returns: Liquidity and Idiosyncratic Risk. *Working Paper*, 05-13.
51. Sun, K., Wang, H. and Zhu, Y. (2021). What Drives the Tail Risk Effect in the Chinese Stock Market?. Available at SSRN 3851205.
52. Tanabe, K. (2018). Pareto's 80/20 rule and the Gaussian distribution. *Physica A: Statistical Mechanics and its Applications*, 510, pp. 635-640. https://doi.org/10.1016/j.physa.2018.07.023.