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Research Article

A new approach for ranking decision-making units in data envelopment analysis by using communication game theory

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Abstract

Ranking decision making units (DMUs) is an important topic in data envelopment analysis (DEA). When efficient DMUs or inefficient DMUs have the same efficiency score, the traditional DEA model usually fails to rank all DMUs. For the sake of comparing and improving the discrimination power of DMUs, some proposed approaches use cooperative game theory for ranking DMUs. In this paper, communication game theory, which includes a transferable utility cooperative game and an undirected graph describing limited cooperation between players, can be used to rank DMUs. The idea

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is that the ranking of DMUs can be done by measuring the effect of removing a subset of DMUs on the total share of the remaining DMUs obtained by the reference frontier share model. In the proposed approach, the players are the DMUs, and the characteristic function measures the increase and decrease in the total share of each DMU. The current paper considers the total share for efficient and inefficient DMUs to rank all DMUs. The proposed approach has been tested on several datasets and compared with the results of the previous ranking methods, which sometimes coincide. In the empirical study, a complete ranking of DMUs is useful and reasonable.

AMS subject classifications (2020): Primary 45D05; Secondary 42C10, 65G99.

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1 Introduction

Data envelopment analysis (DEA) is a popular nonparametric approach in operations research and economics to evaluate the relative efficiency of a group of homogeneous decision-making units (DMUs) that consume multiple inputs to produce multiple outputs. Charnes, Cooper, and Rhodes [11] proposed the first DEA model under the assumption of constant returns to scale (CRS) technology. DEA traditional models can define the efficiency score for each DMU as the sum of weighted outputs divided by the sum of weighted inputs. The efficiency score is constrained to lie between zero and one, and if it is equal to one, then it means that the evaluated DMU is efficient, and if it is less than one, then the evaluated DMU is inefficient.

The traditional DEA models evaluate many DMUs as efficient, but they are not able to attain ranking for these units. The lack of discrimination power among the efficient DMUs leads to the major drawbacks of the traditional DEA models. Some scholars have proposed some new approaches for improving the discrimination power of the traditional DEA model, among them, we focus on the reference frontier share model proposed by Rezaeiani and Foroughi [42]. They defined the concept of the reference frontier to rank the efficient DMUs. The reference frontier of an inefficient DMU is the set of all efficient DMUs that can make better the efficiency of that inefficient DMU. Their model measures the maximum value of the efficient DMU coefficient in all convex combinations of the reference frontier points of inefficient DMUs. The importance of each efficient DMU is specified by these values, which can be defined as the share of each efficient DMU in the creation of the reference frontiers for inefficient DMUs.

In classical transferable utility (TU) cooperative game theory, it is generally assumed that all players can form a cooperation coalition and that there are no limitations on the communication and cooperation possibilities among the players [9]. The presence or absence of restrictions in communication and cooperation possibilities between players may also cause or prevent their communication and cooperation. In many practical situations, there may be limited cooperation and communication between players because of certain social, economic, hierarchical, or technical structures [23]. Several models have been proposed and studied for limited cooperation and communication situations. Among them, we focus on the communication game introduced by Myerson [35]. A communication game consists of the TU cooperative game and an undirected graph in which vertices and links are the players and direct bilateral communication channels between players, respectively; see [26].

This paper first reviews the literature to explain the various proposed approaches to solve the major drawbacks of the traditional DEA model. Next, we present the ranking problem of DMUs in DEA from a communication game perspective, which is cooperative games enriched by a communication structure. The idea of the proposed approach is that if some efficient DMUs were not present, then the total share of other efficient DMUs would increase or decrease, and even some inefficient DMUs could become efficient. If some inefficient DMUs were not present, then the total share of efficient DMUs would decrease. The total share of an efficient DMU is the sum of the reference frontier share of that efficient DMU for each inefficient DMU. The total share of an inefficient DMU is the sum of the reference frontier share of each efficient DMU for that inefficient DMU.

The contribution of an efficient DMU can be measured by increasing or decreasing the total share of other efficient DMUs when this efficient unit is removed from the sample. The contribution of an inefficient DMU can be measured by decreasing the total share of the efficient DMUs when this inefficient unit is removed from the sample. This means that an important efficient DMU is one in that the total share of other efficient DMUs increases by a large amount, and its total share decreases by a large amount whenever it is dropped from the sample. The important inefficient DMU is one that the total share of efficient DMUs decreases by a large amount whenever it is dropped from the sample. However, in the end, the total share of efficient and inefficient DMUs is collectively obtained by defining the frontier set of inefficient and efficient DMUs, respectively. Thus, we can extend the removal of a DMU to the removal of the different subsets of DMUs in the sample. The production possibility set (PPS) is derived collectively from DMUs; thus, the ranking DMUs can be considered a cooperative game. Since the efficiency of each inefficient DMU can only be improved by its reference frontier DMUs and the efficient DMUs can impact the shape of the efficient frontier, there are two cooperation and communication possibilities between inefficient units and their reference frontier units, and among two efficient DMUs that directly linked in the efficient frontier. Therefore, we define the limited TU cooperative game, called the communication game, to rank all DMUs. For TU communication games with a graph structure, a well-known allocation rule is the Myerson value. In the proposed game, the players are the DMUs, and the characteristic function measures the increase and decrease in the total share of each DMU.

DMUs cooperate in sharing their information to maximize efficiency and reach a win-win situation. For this purpose, cooperative games allow for calculating the maximum efficiency of any possible coalition. In DEA, efficient DMUs are the reference frontier for inefficient DMUs to improve their efficiency. In other words, the efficiency of inefficient DMUs is improved by their reference frontier, and efficient DMUs only improve the efficiency of their improved frontier. For this purpose, communication games provide to maximize efficiency in a situation where there is limited communication between efficient and inefficient DMUs. In some cases, the results of the ranking DMUs have been consistent or different for certain data sets, which is not unusual. Each ranking approach has a logic that gives it validity. The logic of our approach is defined based on the communication games that the cooperation of DMUs in defining the reference frontier and the Improved frontier gives validity to the approach. Therefore, our proposed method can provide acceptable results as a ranking approach.

The structure of this paper is as follows. In Section 2, five major DEA ranking approaches are reviewed. Section 3 briefly presents some preliminaries. Section 4 presents the approach proposed for ranking the DMUs. Some numerical examples related to datasets taken from the literature are reported in Section 5. Finally, we summarize and conclude in Section 6.

2 Literature review

Several approaches have been proposed to rank DMUs in DEA. Among them, we can refer to five major groups of approaches. The first group of DEA ranking approaches is those based on cross-efficiency. The cross-efficiency method evaluates the efficiency scores of each DMU by its favorable weights and the favorable weights of other DMUs obtained by the CRS multiplier DEA formulation. The average of these efficiency scores is the cross-efficiency scores of this DMU [44]. Many alternative secondary goals have been proposed because of the existence of alternative optimal weights. The first secondary goal models proposed by Doyle and Green [13] are the benevolent and aggressive strategic models. The benevolent and aggressive strategic models, respectively, maximize and minimize the cross-efficiency of other DMUs simultaneously while keeping their score optima. Liang et al. [29, 30] developed benevolent and aggressive secondary target models using slack variables for solving nonlinear models proposed by Doyle and Green. Wang and Chin [48] maximized the efficiency of each output of the DMU while keeping the efficiency of all DMUs at the maximum efficiency level. Subsequently, this model was extended by Wang, Chin, and Jiang [49] to maximize the efficiency of each input of the DMU simultaneously. The peer-restricted cross-efficiency model was proposed by Ramon, Ruiz, and Sirvent [39] that limits the differences among weights and removes variables with zero weights. Wang, Chin, and Luo[50] first used a virtual DMU in the cross-efficiency model and then increased and reduced the gaps between the weighted inputs or outputs associated with the evaluated DMU and the virtual DMU. Scholars have also presented new ideas for secondary goal models [12, 54, 25, 32]. Table 1 lists a number of representative methods of this group.

Table 1: Some ranking methods based on cross-efficiency approaches.

Reference	Remarks
sexton, Silkman, and Hogan [44]	Cross-efficiency
Doyle and Green [13]	Secondary goals (aggressive/benevolent formulation)
Liang et al. [29, 30]	Extend the model of Doyle and Green
Wang [48]	New alternative models
Wang, Chin, and Jiang [49]	Simultaneously input- and output-oriented weight
Ramon, Ruiz, and Sirvent [39]	Select of the profiles of weights
Wang, Chin, and Luo [50]	A virtual ideal DMU and a virtual anti-ideal DMU

The second group corresponds to methods based on super-efficiency. Andersen and Petersen [4] introduced (AP) the super-efficiency for ranking extreme efficient DMUs. AP model drops the DMU under evaluation from the set of DMUs and then obtains a super efficiency score for this DMU. A major concern related to the AP models is the crucial infeasibility issue. Scholars have also proposed several models for solving the infeasibility problem of envelopment models or unboundedness of multiplier models [52, 45, 34]. Table 2 lists a number of representative methods of this group.

Table 2: Some ranking methods based on super-efficiency approaches.

Reference	Remarks
Andersen and Petersen [4]	Radial, infeasibility issues for VRS
Ma et al. [34]	Calculate more reliably by super efficiency
Xie et al. [52]	The efficiency of thermal power plants
Sojoodi, Dastmalchi, and Neshat [45]	Two super-efficiency models

A third major group of DEA ranking approaches is those based on reference-based approaches. This method defines references for all inefficient to determine the importance of efficient DMUs and improve the ranking of efficient DMUs. Bergendahl [7] developed the DEA model to evaluate the efficiency of a single bank and formed a reference set of banks, which is a convex combination of efficient banks. Boljuncic [8] assessed the changes of inputs and outputs of an extreme efficient DMU thus obtained the region of efficiency for this DMU. Roshdi, Mehdiloozad, and Margaritis [43] used the least distance projection to determine the maximal closest reference set, which is the set of all possible reference DMUs associated with a given closest projection. Rezaeiani and Foroughi [42] introduced a reference frontier model to discriminate among efficient DMUs. This model measures the reference frontier share of each efficient DMU in improving the efficiency of inefficient DMU. Scholars have also presented new ideas for reference-based models [6, 5, 2]. Table 3 lists a number of representative methods of this group.

Table 3: Some ranking methods based on reference-based approaches.

Reference	Remarks
Bergendahl [7]	Use form a reference bank
Boljuncic [8]	Assess changes in an extreme efficient DMU
Roshdi, Mehdiloozad, and Margaritis [43]	The maximal closest reference set
Rezaeiani and Foroughi [42]	The reference frontier share of efficient DMUs
Behmanesh, Rahimi, and Gandomi [6]	test evolutionary many-objective algorithms

The fourth major group of DEA ranking approaches is those based on cross-influence. The cross-influence method excludes certain subsets of DMUs from the sample that defines the technology and then measures the impact on all the DMUs. A ranking model for extreme efficient DMUs was proposed by Jahanshahloo et al. [22]. This model first excludes the efficient DMUs from a set of DMUs and then forms the new efficient frontier. This new frontier gets closer to the inefficient DMUs, and some inefficient DMUs even become efficient. Du et al. [14] introduced an approach for ranking all DMUs based on two approaches of the influence of each efficient DMU on all the other DMUs and the standard efficiency scores. Hosseinzadeh Lotfi et al. [20] proposed a ranking approach based on applying aggregate units, which are artificial and are defined over efficient DMUs. This method first deletes one efficient DMU with better performance and then selects a set of input and output weights that can maximize the group efficiency once again. Izadikhah and Farzipoor [21] proposed a new model to fully rank all DMUs. Their model first defines the concept of virtual DMU as an average of all inefficient DMUs and then measures the impact of efficient DMUs on virtual DMU and the impact of efficient DMUs on the influences of other efficient DMUs. Scholars have also presented new ideas for cross-influence models [40, 15, 18]. Table 4 lists a number of representative methods of this group.

Table 4: Some ranking methods based on cross-influence approaches.

Reference	Remarks
Jahanshahloo et al. [22]	Omission of efficient DMUs from reference set
Du et al. [14]	The influence of efficient DMUs on other DMUs
Hosseinzadeh Lotfi et al. [20]	Deletion of an efficient DMU on another ones
Izadikhah and Farzipoor [21]	Single virtual inefficient DMU
Ramon, Ruiz, and Sirvent [40]	The benchmarking of DMUs

The last major group of DEA ranking approaches is those based on game theory. In recent years, mechanisms of cooperation and non-cooperation in the DEA have attracted increasing attention. Nakabayashi and Tone [36] introduced a model based on DEA and the game theory approach for the consensus-making problems between individuals or organizations with multiple criteria to allocate and impute the given benefit or cost to the players. Wu et al. [51] proposed a method based on the Nakabayashi and Tone approach and cross-efficiency DEA model and obtained the common set of weights for fully ranking DMUs. Liang et al. [31] generalized the original DEA cross-efficiency concept to noncooperative game cross-efficiency. In their game model, each DMU (player) maximizes its cross-efficiency score when the cross-efficiency scores of each of the other DMUs do not deteriorate. The obtained game cross-efficiency scores were unique and constituted a Nash equilibrium point. Abing et al. [1] proposed an approach based on the multi-objective DEA model and Shapley value to solve the problem of many inputs and outputs and the small number of DMUs. Omrani, Beiragh, and Kaleibari [37] proposed the principal component analysis DEA model with the cooperative game for evaluating the performance of Iranian electricity distribution companies to increase the discriminant power of the DEA model with many inputs and outputs. Rezaee, Izadbakhsh, and Yousef [41] introduced a cooperative game theory approach based on the DEA and Nash bargaining game for evaluating the performance of transportation systems by a large scale of measures. The first ranking problem of the efficient DMUs based on cooperative games and DEA was proposed by Li et al. [27]. They first dropped a nonempty subset of efficient DMUs from the set of DMUs and then measured the impact of its removal on the super-efficiency scores of the efficient units. Hinojosa et al. [19] proposed a new cooperative game approach in which first a nonempty subset of efficient DMUs from the set of DMUs dropped and then measured the impact of its removal on the efficiency scores of the inefficient DMUs. In Li et al. and Lozano et al. models, the set of players is the set of efficient DMUs. Omrani, Fahimi, and Mahmoodi [38] applied the game theory approach to increase distinguish power in the DEA model and obtained the fair weights in the cross-efficiency DEA context. Sun, Li, and Wang [47] proposed a comprehensive model based on game theory and the DEA model to improve resource utilization efficiencies and reduce pollutant emissions in the circular economic system. Soofizadeh and Fallahnejad [46] used the theory of cooperative games and the Shapley value method as a fair method to solve real practical problems with imprecise dates and rank airline groups. Their ranking for groups is based on the average marginal shares of groups in different coalitions. Fallahnejad, Asadi Rahmati, and Moradipour [16] used the Shannon entropy to create a new Shapley Vvalue obtained from aggregating the marginal effects of efficient DMUs weighted for ranking efficient DMUs. An et al. [3] proposed a cooperative social network partition approach by using DEA and game theory. The relationship among DMUs is defined by measuring the increased payoff of two cooperative DMUs. They employed the Shapley value to achieve community partition. Ghaeminasab et al. [17] proposed a coalition game to solve the revenue allocation problem by considering cooperative relations among DMUs. They used the concept of DEA efficiency, a new characteristic function, and the Shapley value. Chang, Lin, and Ouennich [10] applied the first DEA-based Nash bargaining models to help acquirer companies and obtain their most desired target companies. Lozano [33] proposed a bargaining model where players are input or output variables for each DMU. Players try to reduce an input variable or an undesirable output and increase a desirable output. Yu and Rakshit [53] computed the input and output targets using the bargaining approach and DEA for a sample of major global airlines. This model performs an unbiased, rational negotiation between the inputs and outputs to achieve reasonable and optimal solutions for inputs and outputs. Table 5 lists a number of representative methods of this group.

Table 5: Some ranking methods based on game theory.

Reference	Remarks
Nakabayashi and Tone [36]	Solve egoists dilemma using cooperative game
Wu et al. [51]	Bargaining game model to evaluate DMUs
Liang et al. [31]	The DEA game cross-efficiency model
Abing et al. [1]	Shapley value-based multi-objective DEA
Omrani, Beiragh, and Kaleibari [37]	DEA cooperative game
Jahangoshai, Izadbakhsh, and Yousef [41]	DEA-game for ranking of operational
Li et al. [27]	Super-efficiency evaluation by a cooperative game
Hinojosa et al. [19]	Ranking efficient DMUs using cooperative game
Omrani, Fahimi, and Mahmoodi [38]	Game theory to find weights in cross-efficiency
Sun, Li, and Wang [47]	A game meta-frontier DEA approach
Soofizadeh and Fallahnejad [46]	Evaluation by using cooperative game fuzzy DEA
Fallahnejad, Asadi Rahmati, and Moradipour [16]	An entropy based Shapley value to rank
An et al. [3]	Cooperative social network community partition
Ghaeminasab et al. [17]	The Coalitional Game and DEA
Chang, Lin, and Ouennich [10]	DEA-based Nash bargaining approach
Lozano [33]	Bargaining approach for efficiency assessment
Yu and Rakshit [53]	The DEA bargaining approach

In recent studies, it is generally considered that players can form a cooperation coalition and there are no limitations on the communication and cooperation possibilities among them. The purpose of this paper is to propose a model from a perspective of limited communication situations to rank all DMUs. The basic assumption in this paper is that there are limitations on the communication and cooperation possibilities among the players. In other words, we develop a ranking model of the DMUs through the DEA reference frontier share model and using TU game theory ideas with restrictions in communication and cooperation. Based on these, we define the communication game and use the Myerson value defined as the Shapley value of the modified game.

3 Preliminaries

The TU cooperative games describe situations where players can form coalitions and payoffs of coalitions can be freely distributed among its members. Mathematically, a TU cooperative game is a pair (N, v), where N is a finite set of players and $v : 2^N \to R$ is a characteristic function with $v(\emptyset) = 0$. A subset $S \subseteq N$ called a coalition. For a subset $S \subseteq N$, the real number $v(S)$ represents the worth of the coalition S.

A value is a function f that assigns to every cooperative game (N, v), a subset $f(v)$ of R^n is called an allocation rule. The most well-known allocation rule that distributes the worth of cooperation among players in TU cooperative game theory is the Shapley value. The Shapley value of the cooperative game for all $i \in N$ is defined by

$$\psi_i(N, v) = \sum_{S \subset N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \, v(S \cup \{i\}) - v(S), \tag{1}$$

where n and s are the cardinality of N and S, respectively.

The Shapley value is the unique allocation rule that satisfies the following four appealing properties:

Efficiency. This property requires that the worth of coalition N is fully distributed among the n players. Mathematically, for every game (N, v), we have

$$\sum_{i \in N} \psi_i(N, v) = v(N).$$

Symmetry. This property demands that two symmetric players get the same payoff. Mathematically, for every game (N, v) and every pair (i, j) of symmetric players, we have

$$\psi_i(N, v) = \psi_j(N, v).$$

Null Player Property. For every game (N, v) and every null player $i \in N$, we have

$$\psi_i(N, v) = 0.$$

Additivity. This property requires the allocation rule to be additive. For every pair of games (N, v) and (N, w), we have

$$\psi(N, v + w) = \psi(N, v) + \psi(N, w).$$

The existence of communication and cooperation restrictions between players in cooperative game theory led to the introduction of new theoretical models commonly referred to as communication games. Mathematically, a communication game is denoted by the triplet (N, v, g), where (N, v) is a TU cooperative game and (N, g) is a graph on N describing the cooperation and communication possibilities between players. In a graph on N, N is a set of vertices and links are direct bilateral communication channels between players. Let us consider a graph (N, g) and two players $i, j \in S$. A link ij exists in the graph g if and only if the players i and j can directly communicate. For any $i, j \in S$, i and j are connected in S if and only if $i = j$ or there are $i = i_0$, i_1, ..., $i_k = j$ such that $\{i_t, i_{t+1}\} \in g$ for $0 \leq t < k$, namely $i = j$ or i and j are directly linked or indirectly linked in S. If each pair of players in S is connected, then S is connected. A maximal (not necessarily maximum) connected subset of S is called a component of S in g, which is called a component simply if $S = N$. We denote by $N|_g$ and $S|_g$ the set of all components belonging to N in g and the set of all components belonging to S in g, respectively.

The most well-known allocation rule for TU communication games is the Myerson value. Myerson defined the restricted characteristic function v^g : $2^N \to R$ associated with the communication game (v, g) as follows:

$$v^g(S) = \sum_{C \in S|_g} v(C), \qquad \text{for all } S \subseteq N. \tag{2}$$

The interpretation of the restricted characteristic function v^g can be that if $S \subseteq N$ is connected, then players in coalition S can communicate and obtain their initial payoff $v(S)$. If S is nonconnected, then players can only communicate with members of the same connected component and receive the sum of the worth obtained by that component. If g is complete, then every coalition $S \subseteq N$ is connected, and v^g coincides with the initial game v.

The Myerson value of a communication game in TU communication game is a vector $M(N, v) \in \Re^N$ for all $i \in N$ defined by

$$M_i(N, v, g) = \sum_{S \subset N \setminus \{i\}} \frac{s!(n-s-1)!}{n!}\ v^g(S \cup \{i\}) - v^g(S). \tag{3}$$

The Myerson value is the Shapley value of the TU communication game in which the worth of each coalition equals the sum of the worth of each connected component belonging to the coalition. The Myerson value can be uniquely determined by component efficiency and fairness properties.

Component efficiency. This property requires that the payoff of each coalition, corresponding to one of the connected components of the communication game is fully distributed among its members. Mathematically, for every communication game and component $C \in N|_g$, we have

$$\sum_{i \in C} M_i(N, v, g) = v(C).$$

The component efficiency property coincides exactly with the efficiency property of Shapley value if the graph g is connected.

Fairness. This property requires that the removal of a link in the TU communication game leads to changes in the payoffs of both players by the same amount. Mathematically, for every communication game (N, v, g) and every link $(i, j) \in g$, we have

$$M_i(N, v, g) - M_i(N, v, g \setminus ij) = M_j(N, v, g) - M_j(N, v, g \setminus ij).$$

We will define the TU communication game and use the Myerson value as the allocation rule to rank DMUs by using a DEA reference frontier share model. The DEA reference frontier share model is defined as follows. Suppose for a sample with m independent DMUs and $j \in M = \{1, 2, \ldots, m\}$ that each DMU consumes k inputs x_{ij} $(i \in I = \{1, 2, \ldots, k\})$ to generate h outputs $y_{rj}(r \in H = \{1, 2, \ldots, h\})$. Let DMU_o be a member of the set of efficient DMUs and let $DMUp$ be a member of the set of inefficient DMUs. The reference frontier of DMU_p, denoted by N_p, is all possible efficient DMUs for DMU_p that dominate it. The reference frontier share of an efficient DMU_o for inefficient DMU_p is defined as the maximum value of the coefficient associated with DMU_o in all convex combinations of reference frontier points of DMU_p. The reference frontier share of DMU_o for DMU_p, denoted by λ_{po}, is defined as follows:

$$\lambda_{po} = \max \{\lambda_o | (\sum_{j\in E} \lambda_j x_j, \sum_{j\in E} \lambda_j y_j) \in N_p \quad \sum_{j\in E} \lambda_j = 1, \lambda_j \geq 0, \quad (j \in E)\}. \tag{4}$$

Let E and IE be the set of efficient and inefficient DMUs, respectively. The total share of DMU_o is defined by $\overline{\lambda}_o = \sum_{p\in IE} \lambda_{po}$, requiring that IE is nonempty. The reference frontier share model for ranking efficient DMUs is as follows:

$$\begin{array}{lll}
\lambda_{po}(M) = \max \ \lambda_o & & \\
\text{s.t.} & \sum_{j\in E} \lambda_j x_j \leq x_{ip}, & i = 1,\dots,m, \\
& \sum_{j\in E} \lambda_j y_j \geq y_{rp}, & r = 1,\dots,s, \\
& \sum_{j\in E} \lambda_j = 1, & \\
& \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} + u_o = d_j, & j \in E, \\
& \lambda_j \leq t_j, & j \in E, \\
& d_j \leq M'(1-t_j), & j \in E, \\
& v_i \geq 1, & i = 1,\dots,m, \\
& u_r \geq 1, & r = 1,\dots,s, \\
& u_o \ \ free, & \\
& \lambda_j \geq 0 \ \ d_j \geq 0 \ \ t_j \in \{0,1\}, & j \in E.
\end{array} \tag{5}$$

The constraint $\sum_{j\in E} \lambda_j = 1$ assures us that for some DMU_j, we have $\lambda_j > 0$. Hence, for these DMU_j, we have $t_j = 1$ and $d_j = 0$. For each $T \subseteq M$, let $\lambda_{po}(T)$ be the reference frontier share of the DMU_o for DMU_p when the sample of DMUs under evaluation is the DMUs in T. Rezaeiani and Foroughi proved that model (5) is always feasible and includes an efficient point of the PPS. In their model, M' is a sufficiently large positive number.

The first ranking approach of efficient DMUs based on cooperative game theory is proposed by Li et al. [27]. In their model, players are the efficient DMUs, and the characteristic function measures the changes in the super-efficiency of the efficient DMUs belonging to S. Formally, the proposed game of Li et al. is a cooperative TU game (N, v), where the characteristic function, for each $S \subseteq N$, is $v(s) = \sum_{k \in N} E_k(M \setminus S) - 1$. Next, Hinojosa et al. [19] introduced a new TU cooperative game approach. In this game, the characteristic function measures the changes in the efficiency scores of the inefficient DMUs when a given coalition of efficient units is dropped from the set of DMUs. Formally, the game defined by Lozano et al. is a cooperative TU game (N, v), where the characteristic function for each $S \subset N$ is $v(S) = \sum_{j \in M \setminus N} E_j(M \setminus S) - E_j(M)$. Moreover, $E_j(T)$ is the efficiency score of the $DMU_j \in T$ obtained by the input-oriented CCR model when the sample includes the DMUs belonging to T.

4 The proposed approach

In the present paper, the total share is proposed for each of the inefficient and efficient DMUs to increase the discrimination power among DMUs. We define the concept of the reference frontier share for inefficient DMUs. We concentrate on the total share of DMUs that are removed from the sample and efficient DMUs outside coalition S; that is, the role of the deleted units in changing the total share of other DMUs and the role of the remaining efficient units in improving the efficiency of the inefficient DMUs in the modified sample are considered. A TU communication game approach is proposed in which the characteristic function measures the effect of removing a subset of DMUs on the total share of all DMUs. This communication game consists of a TU cooperative game and an undirected graph in which the vertices are the DMUs. In the proposed graph, a link represents a direct bilateral communication channel between an inefficient DMU and its reference DMU and among two efficient DMUs that are directly linked in the efficient frontier.

The reference frontier of an inefficient DMU is the set of all efficient DMUs of the efficient frontier that dominate it. The improved frontier of an efficient DMU is the set of all inefficient DMUs of observed DMUs that this efficient DMU dominates them. Each point of the reference frontier of an inefficient DMU can improve that inefficient DMU and each point of the improved frontier of an efficient DMU can be improved by that efficient DMU. The reference frontier share of an efficient DMU is defined as the maximum value of the efficient DMU coefficient in all convex combinations of the reference frontier points of an inefficient DMU. The reference frontier share of efficient DMUs for inefficient DMUs is presented in a matrix. In this matrix, each column corresponds to the efficient DMUs, and each row corresponds to all DMUs. The total share of the efficient DMU is the sum of the reference frontier share of that efficient DMU for its improved frontier points. This share is equal to the sum of the values in the column associated with that DMU. The total share of the inefficient DMU is the sum of the reference frontier share of its reference frontier points for that inefficient DMU. This share is equal to the sum of the values in the row associated with that DMU. The closer an inefficient unit is to the efficient frontier, the less efficient units are involved in improving its efficiency, and the reference frontier of this inefficient unit is smaller. As a result, the reference frontier share of this inefficient unit is lower. Therefore, the lower the reference frontier share of an inefficient unit, the higher the efficiency of that inefficient unit than other inefficient units.

Since the reference frontier share is defined by efficient and inefficient DMUs, removing an efficient or inefficient DMU leads to changes in the total share for each DMU. When the efficient DMU is eliminated, the total share of other efficient DMUs would decrease or increase. If the weak efficient or inefficient DMUs become extreme efficient by removing one efficient DMU, then the total share of the efficient DMUs would decrease. The total share of efficient DMUs would increase when the weak efficient or inefficient DMUs do not become extreme efficient by removing one efficient DMU. Let DMU_o be dropped from the original sample M, $S = \{DMU_o\}$, let the inequality $\overline{\lambda}_j(M) \leq \overline{\lambda}_j(M \setminus S)$ or $\overline{\lambda}_j(M) \geq \overline{\lambda}_j\ (M \setminus S)$ hold for efficient $DMU_j \in M \setminus \{DMUo\}$, and let $\overline{\lambda}_o(M) \geq \overline{\lambda}_o(M \setminus S)$. When the inefficient DMU is eliminated, the total share of efficient units would decrease. If DMU_p is dropped from M, $S = \{DMU_p\}$, then the inequality $\overline{\lambda}_j(M) \geq \overline{\lambda}_j(M \setminus S)$ holds for each efficient DMU. Therefore, if only a certain subset of the efficient DMUs is dropped from the set of DMUs, then the total share of the deleted efficient DMUs for all inefficient DMUs decreases by an amount equal to $\overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S)$ and the total share of the remaining efficient DMUs would increase or decrease by an amount equal to $\overline{\lambda}_j(M \setminus S) - \overline{\lambda}_j(M)$ or

$\overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S)$. If only a certain subset of the inefficient DMUs is dropped from the set of DMUs, then the total share of efficient DMUs decreases by an amount equal to $\overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S)$. The total share of DMUs in $M \setminus S$ decreases by an amount equal to $\overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S)$ or increases by an amount equal to $\overline{\lambda}_j(M \setminus S) - \overline{\lambda}_j(M)$ when a certain subset of the efficient and inefficient DMUs is dropped from the set of DMUs. Moreover, $\overline{\lambda}_j(M \setminus S) - \overline{\lambda}_j(M)$ is the increment on the total share that jth DMU enjoys when the DMUs in coalition S drop from the sample. The difference $\overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S)$ is the reduction on the total share that DMU_j suffers when the DMUs in coalition S drop from the sample.

Therefore, we can define a cooperative TU game (N, v) in which the players are the DMUs, and the characteristic function for each nonempty coalition $S \subseteq N$ is as follows:

$$v(S) = \sum_{k \in S} \overline{\lambda}_k(M) - \overline{\lambda}_k(M \setminus S) + \sum_{j \in M \setminus (S \cup P)} \overline{\lambda}_j(M) - \overline{\lambda}_j(M \setminus S), \qquad (6)$$

where P is the set of all inefficient DMUs belonging to $M \setminus S$.

Since the reference frontier share model aims to measure the share of each efficient DMU in improving the efficiency of inefficient DMUs and the reference frontier share of efficient units is defined by the reference frontier of inefficient units, the inefficient DMUs and their reference frontier DMUs are invited to the game. The game only runs between efficient and inefficient DMUs. In our proposed approach, there exists a commodity that is the corresponding benchmark credit for all DMUs. This commodity represents the total share by the reference frontier share model, which can be transferred among the players. Since the reference frontier share is defined by the efficient and inefficient DMUs, there is only cooperation between inefficient DMUs and their reference frontier. Therefore, there are communication restrictions in a cooperative game. The allocation rule is defined as distributing the total share among the efficient and inefficient DMUs. We use the well-known allocation rule called Myerson value to rank DMUs. Myerson value is the Shapley value in the modified game (N, v^g) and can be specified by the performance and fairness features of the component. The proposed game on a player set N is a triplet (N, v, g) such that (N, v) is a TU game and

(N, g) is a graph. A graph (N, g) consists of vertices in which the vertices are the DMUs and a link represents a direct bilateral communication channel between inefficient DMU and its reference DMU and among two efficient DMUs that are directly linked in the efficient frontier. The characteristic function is the changes in the total share of the DMUs in the modified sample in which the DMUs belonging to coalition S were removed.

The TU in cooperative game theory assumes that one player can freely transfer part of their utility to another player and the players have a common commodity that is valued equally by all and enjoys the same total utility. In the proposed approach, we consider a commodity that stores utility and which can be transferred among the players. The characteristic function of the grand coalition represents the total share uncovered by the DEA reference frontier share. Thus, the implicit commodity is the benchmark credit for contributing to uncovering the total share present in the observations. It is clear that, since the reference frontier share is defined by efficient and inefficient DMUs, removing an efficient or inefficient DMU leads to changes in the total share for each DMU. These are the DMUs that deserve the credit. The credit allocation obtained with the Myerson value is used to rank the efficient and inefficient DMUs.

The following algorithm expresses the importance of DMUs in our proposed approach:

Step 1. Let $DMU_k \in N$ be as a player and list all possible subsets N without player k.

Step 2. List the reference frontier or improved frontier of DMU_k, define the graph (N, g) and obtain $S|_g$.

Step 3. Solve model (5) four times and obtain $\overline{\lambda}_k(M)$ and $\overline{\lambda}_k(M \setminus S)$ for $k \in S$, and $\overline{\lambda}_j(M)$ and $\overline{\lambda}_j(M \setminus S)$ for $j \in M \setminus (S \cup P)$.

Step 4. Based upon (6), Calculate $v^g(S)$.

Step 5. Calculate the Myerson value of DMU_k on the use of formulations (2) and (3).

Example 1. [42] Table 6 shows the dataset related to eight DMUs with one input and one output. Consider the sample that includes all DMUs, $S = \emptyset$. In this case, DMUs A, B, C, F, and G appear to be efficient. The reference frontier of inefficient DMU_D for $S = \emptyset$ is shown in Figure 1. The reference frontier share model obtains the reference frontier share of the efficient DMUs for all DMUs and is represented as a matrix below:

DMU	A	B	C	F	G
A	1	0	0	0	0
B	0.5	1	0.5	0	0
C	0	0	1	0	0
D	0.75	1	1	0.5	0.333
E	0.25	0.5	1	0	0
F	0	0	0.333	1	0.666
G	0	0	0	0	1
H	0.25	0.5	1	1	0.666

Table 6: Dataset and the total share of all DMUs

DMUs	A	B	C	D	E	F	G	H
x_1	2	3	4	5	4	6	7	6
y_1	3	5	7	4	6	8	8.5	6
$\overline{\lambda}_j(M)$	1.75	2	3.833	3.5833	1.75	1.5	1.667	3.4167
Ranking	3	2	1	8	6	5	4	7

The total share of each efficient DMU is equal to the sum of the values of each column, for example, the result $\overline{\lambda}_A = 1.75$, $\overline{\lambda}_B = 2$, $\overline{\lambda}_C = 3.83$, $\overline{\lambda}_F = 1.5$ and $\overline{\lambda}_G = 1.67$. The more the total share of the efficient DMU, the higher its ranking. The total share of each inefficient DMU is equal to the sum of the values for each row, for example, $\overline{\lambda}_D = 3.584$, $\overline{\lambda}_E = 1.75$ and $\overline{\lambda}_H = 3.417$. The higher the total share of the inefficient DMU, the lower its efficiency. In an undirected graph, vertices are efficient and inefficient DMUs, and links represent direct bilateral communication channels between inefficient DMUs and their reference frontier or efficient DMUs and their improved frontier. To illustrate this graph, we consider two efficient DMU A and inefficient DMU D. Two DMUs A and D are vertices A and D in the graph. The reference frontier of DMU D is $\{A, B, C, F, G\}$, and the improved frontier

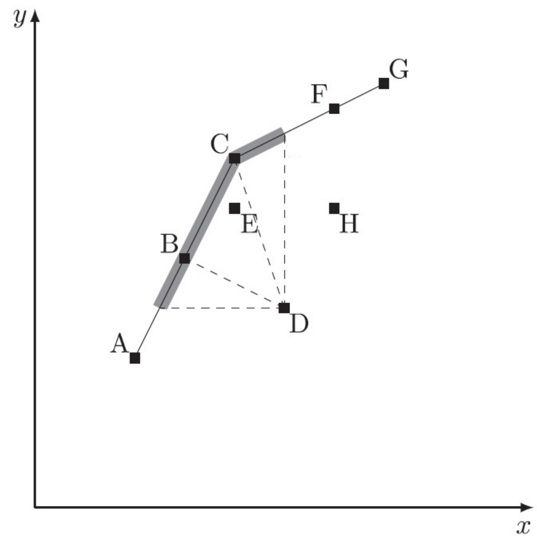

Figure 1: The reference frontier of DMU_D in $S = \emptyset$

of DMU A is $\{B,\ D,\ E,\ F\}$. Therefore, vertices A and D connect to their improved frontier and reference frontier, respectively. The cooperation and communication in the graph (N, g) are shown in Figure 2. According to the graph (N, g) and $S \subseteq N$, the set of all components that belong to S in g, $S|_g$, is defined and then $v^g(S)$ is calculated.

For the coalition $S = \{B, C\}$, the modified sample includes the inefficient DMUs D and H and efficient DMUs A, E, F, and G. Thus, the DMUs outside S coalition define a new efficient frontier. It means that DMUs B and C belonging to the coalition S are not present in the sample. The reference frontier of D inefficient DMU on the modified sample and new efficient frontier is shown in Figure 3.

Let us consider two coalitions $S = \{B, C\}$ and $\overline{S} = \{D, E, F, G, H\}$. The coalition S is connected, $S|g = S$, and therefore players belonging to S obtain their initial worth $v(S)$, that is, $v^g(S) = S$. The coalition $\overline{S}$ is not connected. As shown in Figure 4, $\overline{S}|g = \{\{D, F, G, H\}, \{E\}\}$, and we have $v^g(\overline{S}) = v(\{D, F, G, H\}) + v(\{E\})$.

For the coalition $S = \{B, C\}$, the modified reference frontier share of DMU_A, DMU_E, DMU_F, and DMU_G for inefficient DMU D are 0.8, 1,

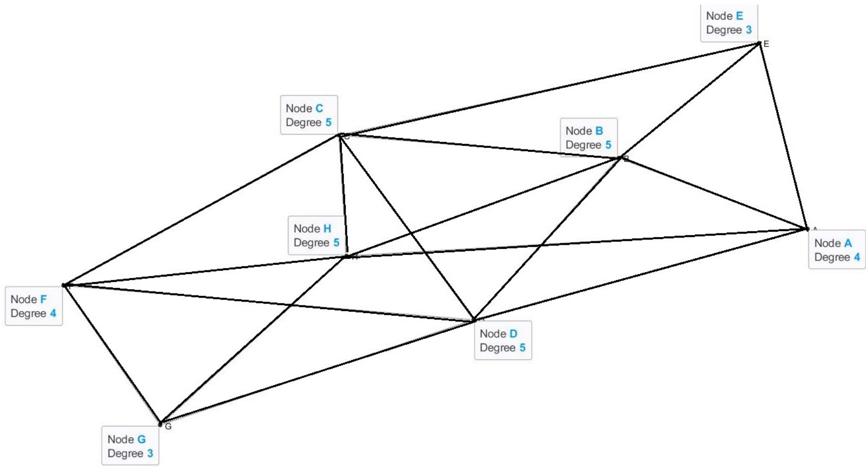

Figure 2: The graph (N, g).

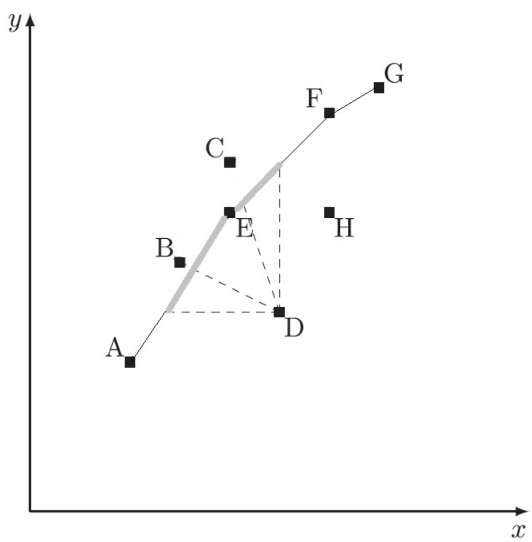

Figure 3: The reference frontier of DMU_D and new efficient frontier for $S = \{B, C\}$ coalition

0.75, and 0.6, respectively, the modified reference frontier share of DMU_A, DMU_E, DMU_F, and DMU_G for inefficient DMU H is 0.4545, 1, 1, and 0.67, respectively. Therefore, the modified reference frontier share of efficient

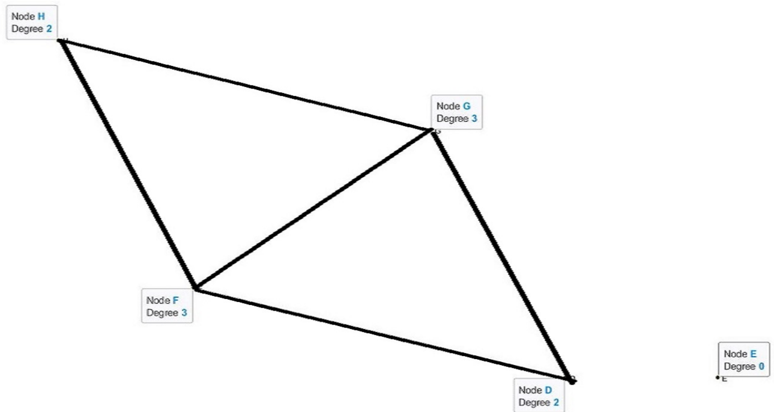

Figure 4: The graph $(\overline{S}, g)$ and the connected components.

DMUs for inefficient DMUs has increased compared to the reference frontier share of the sample $S = \emptyset$. the total share for DMU_A, DMU_D, DMU_E, DMU_F, DMU_G, and DMU_H are 1.2545, 3.15, 2, 1.75, 1.2667, and 3.1212, respectively.

The modified total share has decreased compared to the total share of the sample $S = \emptyset$ because of removing the efficient B and C and appearing DMU_E to be efficient. Therefore, the characteristic value of coalition S in the cooperative game is 4.7255. This value means that removing DMUs B and C from the sample leads to a reduction in the sum of the total share of DMUs. The Myerson value of each DMU and their ranking are shown in Table 7. Efficient and inefficient units are ranked separately. The higher the Myerson value in the efficient units, the more their efficiency. The higher the Myerson value in the inefficient units, the less their efficiency.

Table 7: The Spearman rank correlation coefficient: 0.8095

DMUs	A	B	C	D	E	F	G	H
Myerson value	1.179	1.625	3.259	1.067	2.220	0.189	0.118	4.170
Ranking	3	2	1	6	7	4	5	8
super-efficiency	3	2	1	8	6	5	4	7

5 Numerical illustrations

The proposed approach will be illustrated using a data set from the literature in this section. Then we compare our results with the results obtained by some approaches related to Section 2. We use the Spearman rank correlation coefficient for every four cases.

Example 2. [19] Table 8 shows the dataset related to six DMUs ($M = \{A, B, C, D, E, F\}$) that include two inputs and one output. The total share $\overline{\lambda}_j(M)$ for the DMUs is shown in Table 8. Table 9 shows the result of ranking approaches.

Table 8: Dataset and the total share of all DMUs

DMUs	A	B	C	D	E	F
x_1	13	6	2	1	9	4
x_2	1	3	6	10	5	8
y_1	1	1	1	1	1	1
$\overline{\lambda}_j(M)$	0.4286	1.5000	1.6667	0.5000	2.0952	2.0000
Ranking	4	2	1	3	6	5

Table 9: The proposed ranking of all DMUs. The Spearman rank correlation coefficient: 0.8

DMUs	A	B	C	D	E	F
Myerson value	0.0390	0.8258	1.3425	0.0153	3.2465	2.0235
Ranking	3	2	1	4	6	5
Jahanshahloo	4	2	1	3	–	–

Example 3. [19] We consider the dataset containing eight DMUs listed in Table 10 adapted from Cooper et al. by adding two inefficient DMUs. Results are shown in Table 11.

Table 10: Dataset and the total share of all DMUs

DMUs	A	B	C	D	E	F	G	H
x_1	2	2	5	10	3	10	4	5
x_2	7	12	5	4	6	6	12	11
y_1	3	4	2	2	1	1	2.5	3.5
y_2	1	1	1	1	1	1	1	1
$\overline{\lambda}_j(M)$	2.166	1.8	1.5	1.25	1.25	3.666	3.00	1.3
Ranking	1	2	3	4	5	8	7	6

Table 11: The proposed ranking of all DMUs. The Spearman rank correlation coefficient: 0.6 and 0.7, respectively

DMUs	A	B	C	D	E	F	G	H
Myerson value	1.952	0.814	0.989	0.254	0.199	2.562	3.443	5.372
Ranking	1	3	2	4	5	6	7	8
super-efficiency	1	2	4	3	5	6	8	7
Li (2016)	1	2	3	4	5	–	–	–

Example 4. [24] Table 12 shows the dataset belonging to twelve DMUs with three inputs and two outputs. Moreover, $DMU_4, DMU_5, DMU_6, DMU_8, DMU_9$ and DMU_{10} are efficient DMUs. Table 13 compares the ranking of the proposed approach with other ranking approaches.

Example 5. [28] Table 14 shows the dataset of 18 city commercial bank branches in China with three inputs and three outputs. Feng et al. integrated cooperative game theory and the cross-efficiency method to develop a DEA-game cross-efficiency approach to generate a unique and fair allocation plan. Table 15 compares the ranking of the proposed approach with the DEA-game cross-efficiency approach introduced by Feng et al..

To check the compatibility of the three ranking approaches of Arbitrary, Benevolent, and Aggressive with our approach, we compare their results. By comparing the results of the four ranking methods, we conclude that there are similarities between the three ranking approaches and our approach. The

Table 12: Dataset and the total share of all DMUs

DMUs	x_1	x_2	x_3	y_1	y_2	$\overline{\lambda}_j(M)$	Ranking
1	350	39	9	67	751	3.9	12
2	298	26	8	73	611	2.58	9
3	422	31	7	75	584	3.57	11
4	281	16	9	70	665	3.03	3
5	301	16	6	75	445	3.07	2
6	360	29	17	83	1070	0.92	6
7	540	18	10	72	457	2.36	8
8	276	33	5	74	590	2.42	4
9	323	25	5	75	1074	4.66	1
10	444	64	6	74	1072	2.6	10
11	323	25	5	25	350	1	7
12	444	64	6	104	1199	1.89	5

Table 13: The proposed ranking of all DMUs. The Spearman rank correlation coefficient: 0.8671 and 0.8462, respectively

DMUs	1	2	3	4	5	6	7	8	9	10	11	12
The proposed approach	11	12	9	2	3	6	7	5	1	8	10	4
super-efficiency	10	7	11	3	2	6	8	5	1	9	12	4
Khodabakhshi and Aryavash approach	11	7	9	4	2	6	10	5	1	8	12	3

mean Pearson correlation coefficient of Arbitrary, Benevolent, and Aggressive with our approach equals 0.75439. As a result, this Pearson correlation coefficient confirms the similarity of ranking results between our method and Arbitrary, Benevolent, and Aggressive. Therefore, according to the obtained results, the proposed approach can give an acceptable ranking of DMUs.

In Examples 3 and 4, the Pearson correlation coefficient shows that the ranking approaches coincide with the ranking obtained in our approach. Our approach is the rationale based on the theoretical basis of the reference fron-

Table 14: The dataset of 18 branches of the city commercial bank

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	62	1822	1361	140,117	130,288	5260
2	80	1833	1565	213,774	145,761	10,773
3	129	3595	1378	194,084	130,556	8006
4	62	1978	333	87,876	49,454	4479
5	89	2138	549	107,091	60,872	5897
6	84	1910	704	97,472	94,310	3849
7	36	1234	840	114,001	80,019	5292
8	172	4348	959	366,423	306,926	12,479
9	62	879	1253	107,393	86,485	5132
10	53	2566	483	69,691	43,907	3869
11	92	1348	419	148,458	87,193	7234
12	39	1229	513	83,752	40,046	3984
13	144	4640	1323	223,539	211,466	10,655
14	47	2248	670	70,555	65,110	2205
15	39	1571	362	99,143	66,736	5271
16	56	1635	669	112,513	79,366	5202
17	34	939	867	87,660	56,157	3000
18	58	1807	419	88,334	67,160	4171

tier share model and the communication game theory. Based on the comparison and analysis results of the empirical application, the proposed approach will give a unique and acceptable to all DMUs.

6 Conclusion

Discriminating among all DMUs is an important issue in DEA. The lack of discrimination power between the efficient DMUs leads to the major drawbacks of the traditional DEA models. In this paper, a new approach to DEA is proposed for improving the discrimination power of the traditional DEA model. This is based on the Myerson value, which is a well-known allocation

Table 15: The proposed ranking of all DMUs. The mean Spearman rank correlation coefficient: 0.75439

DMUs	Myerson value	Arbitrary	Benevolent	Aggressive
1	0.8259	0.7631	0.9906	0.6858
2	0.8349	0.8429	0.9998	0.7967
3	0.8130	0.7092	0.9980	0.6491
4	0.8173	0.7366	0.9953	0.7216
5	0.8154	0.7228	0.9974	0.6948
6	0.8104	0.7022	0.9874	0.6412
7	0.8268	0.8251	0.9915	0.7686
8	0.84165	0.8521	1.0000	0.7959
9	0.8267	0.7683	0.9933	0.7209
10	0.8272	0.6861	0.9982	0.6606
11	0.8392	0.8413	0.9941	0.8271
12	0.8239	0.7537	0.9938	0.7025
13	0.8251	0.7534	0.9983	0.7014
14	0.8285	0.6651	0.9870	0.6076
15	0.8375	0.8351	0.9894	0.8114
16	0.8369	0.7732	0.9915	0.7153
17	0.8201	0.7469	0.9922	0.6742
18	0.8183	0.7447	0.9903	0.7047

rule of the communication game theory. The players of the proposed TU communication game are the DMUs. The characteristic function measures the payoff of a coalition in terms of the amount of the total share obtained by the removal of the corresponding DMUs in the modified sample. If some efficient DMUs are not present in the sample, then the total share of other efficient DMUs increases or decreases. If the inefficient DMUs do not cooperate, then the total share of efficient DMUs decreases. As the efficient and inefficient DMUs cooperate and are present in the sample, the modified total share of the efficient DMUs becomes lower and closer to the initial total share and the modified total share of the inefficient DMUs becomes higher and closer to the initial total share. The proposed approach on some datasets from the literature has been tested and used the Spearman rank correlation coefficient for comparison with the results of other methods in each case. Our approach is the rationale based on the theoretical basis of the reference frontier share model and communication game theory. It is based on the participation of each efficient DMU in improving the efficiency of the inefficient DMUs and the participation of each inefficient DMU in creating the reference frontier and defining the total share for the efficient DMUs. Therefore, the proposed approach can be accordingly credited.

As topics for further research, we can consider other TU communication game allocation rules, such as the position value. We can also define the games with particular coalition structures or partition of the set of players and then investigate allocation rules such as Owen value.

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