Journal of Systems Thinking in Practice
JSTINP, *QUARTERLY*
Homepage: jstinp.um.ac.ir
Online ISSN: 2821-1669, Print ISSN: 2980-9460
Research Article
DOI: 10.22067/JSTINP.2024.86349.1089
JSTINP 2024; Vol. 3, No. 2

Exploring Chaos Theory in Economic Growth and Energy Price Dynamics: A Numerical Simulation Approach

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How to cite this article

Jabbari, H., 2024. Exploring Chaos Theory in Economic Growth and Energy Price Dynamics: A Numerical Simulation Approach, *Journal of Systems Thinking in Practice*, *3*(2), pp.93-111. doi: 10.22067/jstinp.2024.86349.1089. URL: https://jstinp.um.ac.ir/article_45273.html

ABSTRACT

Chaos theory offers a unique lens to understand the intricate relationships between economic growth and energy supply pricing. Existing economic theories often emphasize energy prices' inherent randomness, unpredictability, and economic growth. A deeper comprehension can be achieved by applying chaos theory to this complex system. Developing a dynamic model that captures the causal relationships among the various variables impacting economic growth, energy supply, and pricing is crucial for unraveling this complexity. This study aims to delve into the chaotic nature of the energy economy system within the context of economic growth. The research methodology is rooted in a fundamental-applied approach. By employing numerical simulation techniques, specifically utilizing the Simulink MATLAB toolbox, the study seeks to explore and potentially control chaos within the system. The findings highlight the system's nonlinear dynamics, showcasing its sensitivity to initial conditions and exhibiting chaotic behavior, limit cycles, and stable equilibrium points across varying initial values. This research endeavor contributes to a more nuanced understanding of the interplay between energy economics, economic growth, and pricing dynamics.

Keywords

Keywords: Chaos theory, Energy economy, Economic growth, Numerical simulation.

Article history

Received: 2024-01-11
Revised: 2024-05-19
Accepted: 2024-05-29
Published (Online): 2024-06-24

Number of Figures: 6 **Number of Tables: 1** **Number of Pages: 34** **Number of References: 27**

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1. Introduction

Conducted studies and extensive applications of chaos in various sciences, such as electrical engineering, telecommunications, medicine, meteorology, management, and economics, show that this phenomenon and its control are significant. One of the notable reasons for this nonlinear phenomenon is its inherent and individual characteristics, including extreme sensitivity to minimal changes in the initial conditions (Jia, 2007). The research presented on chaos can be divided into two categories in a general classification. The first category includes articles that focus on investigating the nature of the chaos phenomenon, its inherent characteristics, the introduction of new chaos systems, and practical and physical applications of chaos (Jia, 2007; Ge and Yang, 2009; Karawanich and Prommee, 2022). The second category includes articles focusing on chaos control and developing control methods that can be applied to chaotic systems. Carefully, this category of articles explains that one of the main goals of chaos control is to remove the chaotic system from its chaotic state and converge it towards the equilibrium point or a limit cycle (Ott, Grebogi, and Yorke, 1990; Qazza et al., 2023).

On the other hand, studying the behavior of economic, social, and management systems that have high complexity is an important issue (Xue et al., 2023; Banovetz and Oprea, 2023). In the field of energy economy, energy price assessment includes various information on the energy market. Energy prices play an important role in the global economy. With the growth of the economy, the issue of energy prices has attracted widespread global attention. Such issues as the relationship between energy supply, economic growth, and energy price, how to investigate the mechanism of energy price fluctuations, and how to investigate the effects of energy price on economic growth and energy supply have become important topics of academic research. The energy market, control laws, and government policy have played a strategic role in its development. Therefore, optimizing energy price performance indicates the importance of the energy market impact and the better use of control laws and policies to ensure the sustainable development and health of the energy market (Bhadoria and Marwaha, 2023).

Energy price systems, energy supply, and economic growth include many factors, such as energy price, demand, supply, economic growth, and energy efficiency; therefore, they are very complex. How to research this system with nonlinear dynamic models is an important question. Most previous studies on energy prices mainly focused on empirical analysis with a statistical approach and systematic theoretical analysis (Tang and Tan, 2013; Lin and Du, 2014; Bhadoria and Marwaha, 2023; Chen et al., 2022).

At the beginning of the 19th century, mathematicians first noticed chaos theory. In the 1970s, biological researchers applied this theory to population models. In economics, researchers who studied stability and instability in the 1980s and 1990s focused on chaotic systems (Oestreicher, 2007). Zeng et al. (2014) based on energy consumption, carbon emission, and economic growth, researched a three-dimensional system in the Chinese economy. Also, Wang and Tian (2015) modeled the nonlinear dynamic system of economic growth, energy supply, and price in China and investigated the chaotic behavior of the system. Dabachi et al. (2020) investigated the relationships between environmental variables, energy price, energy intensity, and economic growth in OPEC member countries. They suggested that sustainable energy consumption should be considered by adopting appropriate policies. In this research, Pireddu (2023) showed from a mathematical point of view the existence of complex dynamics for a seasonally disturbed version of the Goodwin growth cycle model, both in its initial formulation and for the modified formulation, including the nonlinear terms of the real wage bargaining function and the investment function. The framework of associated convolution maps is introduced by using the periodic dependence on the time of that model parameter. Valid topological results in this field have made it possible to prove that the Poincaré map, related to the systems in question, is disordered, focusing on sets located in the unit field and when dealing with the original version of the Goodwin model. Accordingly, chaotic trademark features such as sensitive dependence on initial conditions and positive topological entropy follow.

Crude oil is the most important source of energy in the world, and oil price fluctuations can significantly affect investors, companies, and governments. The price of crude oil has several characteristics, including randomness, sudden structural changes, inherent nonlinearity, fluctuations, and a chaotic nature. These characteristics make accurate forecasting of crude oil prices challenging. Sun et al. (2022) proposed a hybrid forecasting model for future crude oil prices, whose accuracy and robustness have been shown through controlled experiments and sensitivity analysis. This study uses a new denoising method for data processing to improve crude oil price forecasting accuracy and stability. In addition, chaotic time series forecasting methods, neural networks, linear model forecasting methods, and deep learning methods are adopted as sub-models. The results of interval predictions with low width and high prediction accuracy are obtained by introducing a confidence interval adjustment factor. The results of the simulation experiments showed that the proposed hybrid forecasting model shows higher accuracy and efficiency as well as better robustness in forecasting than the control models. In

 DOI: 10.22067/JSTINP.2024.86349.1089

summary, the proposed forecasting framework can derive accurate point and interval forecasts and provide a valuable reference for future crude oil price forecasting.

Therefore, it is necessary to conduct deeper research on the following aspects. Many factors in the energy market form a complex coupling relationship. For this reason, the energy market needs to clarify the relationships between the factors and then analyze them. On the other hand, there is a huge increase in the emission of greenhouse gases in Iran and the eighty percent share of the energy sector in their emission. Also, the high energy intensity and the lack of use of innovative technologies in industries show the importance of research in this field. In this nonlinear dynamic model, three variables of energy price, energy supply and economic growth are considered as system state variables. State variables are considered auxiliary variables if other factors influence them. This nonlinear dynamic method can analyze this system theoretically and numerically.

This research explores the intricate relationship between energy prices, supply, and economic growth using a novel approach: nonlinear dynamic systems modeling. Unlike conventional studies that primarily focus on linear models, this research delves into the complexities of nonlinear dynamics, emphasizing the importance of stability analysis. By employing the Vensim software environment, a causal model is developed to simulate the complex interactions between variables. This model is further enhanced through a chaos model implemented in MATLAB, providing a deeper understanding of the system's behavior. The research aims to align the energy market with the strategic goal of achieving sustainable economic growth while adapting to evolving energy structures and environmental considerations.

The study's key contributions include: 1) Exploring the stability of nonlinear dynamic systems: This research goes beyond simply modeling the system; it investigates the it's stability, analyzing concepts like stable equilibrium points, chaos, and limit cycles. 2) Utilizing advanced modeling tools: Vensim and MATLAB allow for a comprehensive and robust analysis of the complex relationships within the energy-economy nexus. 3) Providing insights for sustainable development: By understanding the dynamic interplay between energy supply, prices, and economic growth, this research provides valuable insights for developing sustainable energy policies and strategies. This research represents a significant advancement in understanding the energy market dynamics. Incorporating nonlinear dynamic systems theory offers a more accurate and nuanced perspective on the complex relationships within the energy-economy nexus, ultimately contributing to developing more effective and sustainable energy policies.

2. Literature review

In this section, information about linear dynamic systems and non-linear dynamic systems (chaos) is provided.

2.1. Linear dynamic system

The performance of linear systems does not depend on their state and is only related to their initial point and does not depend on their state and position at different times; that is, by having the initial point of movement, it can be known all its future positions. The gradual evolution of linear dynamic systems is also a linear process, and the sum of two solutions in the system is another solution in the system. Also, linear systems can be analyzed by breaking down the problem into smaller components, and then by summarizing the results, they can be analyzed as a whole, and this is one of the things that makes the analysis of linear systems easy. Create (such as Fourier analysis and superposition topics). Finally, the analysis of equations related to these systems is known.

Linear dynamical systems are dynamical systems where the evaluation functions are linear. Dynamical systems generally do not have closed-form solutions, but linear dynamical systems have a rich set of exact mathematical properties. Linear systems can also be used to understand the qualitative behavior of general dynamical systems by calculating the equilibrium points of the system and approximating it as a linear system around each point. Linear dynamical systems can be solved more accurately than nonlinear systems. In addition, any nonlinear system's (approximate) solutions of can be well approximated using an equivalent linear system close to its fixed points (Zhurabok, et al., 2017).

2-2. Nonlinear dynamic systems (chaos)

Chaos comes via Latin chaos from the Greek chaos, "the first state of the universe, the formless state of primordial matter, (personified) the parent of Darkness and Night, infinite and empty space, the expanse of air." The original Greek meaning of cháos was "hole, empty, yawning opening," from an unrecorded cháwos and related to the adjective chaûnos, "loose, spongy, having holes. "The first meaning in English was "an immeasurable and formless void, infinite darkness," especially about the state that preceded God's universe creation. The current meaning, "a state of utter confusion and disorder (resembling the primordial state)" first appeared about 1533 (Kolo and Adepoju, 2015).

Historically, after Newton's laws of motion were presented, many people, relying on the intrinsic certainty of these laws, called them God's calculator and considered them sufficient

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for predicting the future based on current values. In general, it was believed that if the current situation was known with high accuracy, the future could be predicted with the same accuracy. This belief was still in place until the end of the 19th century "Henri Poincaré" in his investigation and effort to solve the three-body problem realized that in some cases, if the accuracy in the initial conditions is high, the uncertainty in the final results is not necessarily insignificant. By reducing the uncertainty in the initial conditions, the uncertainty does not necessarily decrease. This issue was a manifestation of chaotic behavior that was not recognized then. Almost the first numerical research that led to the comprehensive introduction of chaos was presented by "Edward Lorentz" (Sato and Murakami, 1991).

Chaos is a long-term, non-periodic behavior in a deterministic system that shows a sensitive dependence on the initial conditions. The meaning of long-term non-periodic behavior in dynamic systems is that there are paths that do not lead to fixed points, periodic orbits, or quasi-periodic orbits when time tends to infinity. Determinism means that the system does not have stochastic parameters or inputs, but the disordered behavior of these systems is caused by non-linearity. This term is used in contrast to stochastic, which means that the system's behavior is irregular, stochastic, uncertain, and unpredictable. Being sensitive to the initial conditions in dynamic systems means that adjacent paths are rapidly and exponentially separated. This feature is the main difference between chaotic and non-chaotic dynamic systems. In non-chaotic dynamic systems, the initial small difference in two paths is a measurement error. It increases linearly with time, while the difference between two paths with a very small distance increases exponentially in chaotic dynamic systems.

Nonlinear dynamic systems and even discrete linear systems can exhibit completely unpredictable behavior. Such behavior may appear to be stochastic despite the fact that it is essentially certain (i.e., there is no possibility of a random state). This unpredictable behavior is called chaos.

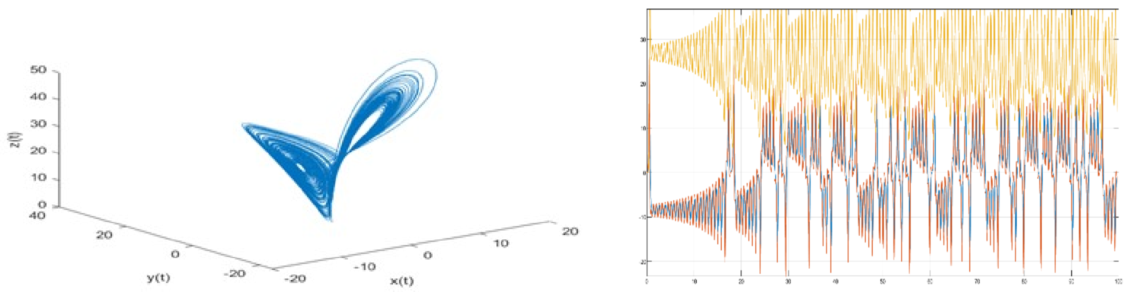
Unlike linear systems, the relationship between speed and position is nonlinear in nonlinear dynamic systems. In such a system, if there are two answers, the sum of them is not the answer of the other system, and the system cannot be divided into smaller components; each can be solved separately, but the whole system must be studied together. Therefore, the equations related to transformation in these systems do not have an analytical solution, or their analytical solution is very difficult (Wang *et al.*, 2019).

One of the characteristics of chaos theory is the butterfly effect introduced by Lorentz. Lorentz published the results of his research in the form of a system of nonlinear differential equations. The calculation results showed the extreme sensitivity of the Lorentz model to the initial values. This property is called the butterfly effect and became the cornerstone of chaos theory. This feature indicates if the variables have a chaotic process. Predicting them in the long run will take much work and effort. In this regard, economic, social, and political systems have a butterfly effect. The butterfly effect is a valid and complete reason for the appropriate and low-cost decisions of a creative and successful manager, which causes a tremendous transformation in his organization.

Lorentz's differential equation system is in the form of equation (1).

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = x(\rho - z) - y \\ \dot{z} = xy - \beta z \end{cases} \quad (1)$$

Lorentz model for initials $(x_0, y_0, z_0) = (0.1,0,0)$, Parameters $(\sigma, \rho, \beta) = (10,28,8.3)$ is simulated in Simulink MATLAB toolbox and the software outputs are shown It will be given. given. In Figure 1. a, the system is drawn in three dimensions. It can be seen that the system has two absorbing points and the continuously moves around these two points from one to the other. Figure 1.b shows three simultaneous systems, and the system can be seen as pseudo-random and chaotic.



a: Three-dimensional Lorentz diagram b: Diagram of three variables of the system

Figure 1. Lorentz diagram and diagram of three variables of the system according to time

Three two-dimensional figures of the system in terms of changes in x, y; x, z and y, z are shown in Figure 2, and in Figure 3, System change is shown separately in time.

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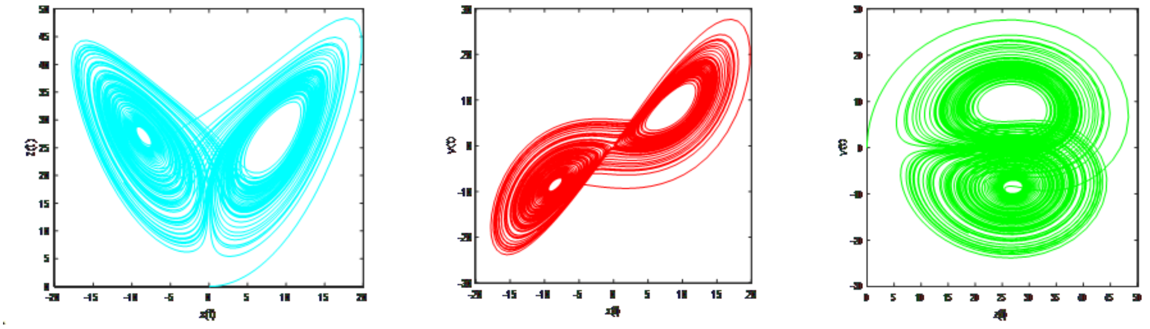

Figure 2. Two-dimensional diagrams of the lorentz

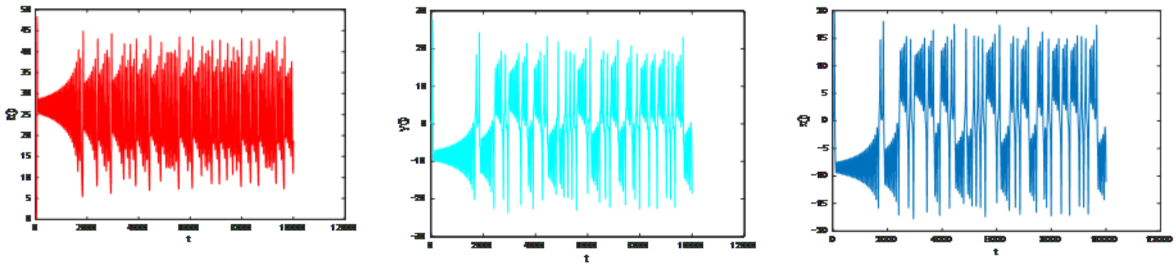

Figure 3. Diagrams of system variables according to time

3. Research method

First, the system's conceptual model is defined based on direct and indirect causal relationships between energy prices, energy supply, and economic growth and the factors affecting them. Then, a non-linear system of energy price dynamics, energy supply, and economic growth is obtained based on the complex relationship between all system factors. Using this system, the dynamic evaluation of these factors was done using numerical simulation. The conceptual model is drawn using Vensim software. Then, the non-linear dynamic system, which has high complexity, is simulated and analyzed by the Simulink MATLAB toolbox.

3.1. Cause and effect model of energy supply system, energy price and economic growth

The studied model drawn in Vensim software includes 71 cause and effect loops, which shows the high complexity of this system. In Figure 4, (adapted from (Wang and Tian, 2015)) "+" indicates positive correlation and "-" indicates negative correlation and $k_i; i = 1,2,\dots,36$indicates the relationship between both elements.

For example, according to Figure 4, energy prices are variable and go up and down; some factors, such as energy consumption and lack of energy supply in the market, will increase energy prices. Increasing energy production will increase energy supply. So, the lack of energy supply is reduced. As a result, the price of energy will also decrease. Meanwhile, with the increase in energy production and consumption in the market, carbon emissions will increase. Then, the government will adjust the energy structure accordingly. These settings will reduce energy intensity and increase energy efficiency. After that, the economy may develop rapidly, increasing energy consumption and increasing energy supply shortages.

Meanwhile, rapid economic development can help promote the advancement of science and technology. Innovative technologies can help us use more energy and increase energy production. Technological development can help exploit new energies, such as solar energy, which can reduce carbon emissions. Similarly, it can improve energy use efficiency and reduce energy intensity effectively; therefore, the economy will develop. As a result, there are some non-linear relationships, direct or indirect, between energy prices, supply, and economic growth.

Therefore, in order to analyze the behavior of the system, it is a system of non-linear differential Equations (2) in terms of state variables depending on the time of energy supply x(t), energy price y(t), and the growth of the gross national product z(t) (measure of size economic growth) and the relationships presented in the cause and effect figure are written; And by using MATLAB software, the model has been simulated and its behavior has been checked.

$$\begin{cases} \dot{x} = a_1 x + a_2(c - y) + a_3(z - d_1) \\ \dot{y} = -b_1 y + b_2 x - b_3 z\left(1 - \frac{z}{d_2}\right) \\ \dot{z} = C_1 z\left(1 - \frac{z}{L}\right) + C_2 yz \end{cases} \quad (2)$$

That,

$$\begin{cases} a_1 = f(k_1 . k_2 . k_3 . k_5 . k_6) \\ a_2 = f(k_1 . k_8 . k_{12} . k_{16} . k_{36} . k_{36}) \\ a_3 = f(k_1 . k_{11} . k_{28} . k_{30} . k_{31} . k_{12}) \\ b_1 = f(k_{21} . k_{22} . k_{23} . k_9) \\ b_2 = f(k_{24} . k_{22} . k_7 . k_{14} . k_{17} . k_{20} . k_{24}) \\ b_3 = f(k_{11} . k_9 . k_3 . k_{18} . k_{17} . k_{24} . k_{20} . k_{23} . k_{28} . k_{29} . k_{27} . k_{34} . k_{22} . k_{21} . k_{33}) \\ c_1 = f(k_{10} . k_{11} . k_{13} . k_{25} . k_{16} . k_{34} . k_{28} . k_{17} . k_{18} . k_{30} . k_{31}) \\ c_2 = f(k_{13} . k_{14} . k_{15} . k_{17} . k_{18} . k_{20} . k_{26} . k_{32} . k_{35} . k_{31} . k_{36}) \end{cases} \quad (3)$$

In differential equations device (3):

a_i ; $i = 1,2,3$: parameters related to energy supply x(t)

parameters related to energy price y(t): b_i ; $i = 1,2,3$

c_i ; $i = 1,2,3$: parameters related to the growth of the national gross product z(t)

k_i ; $i = 1,2,\ldots,36$: Relationship between variables

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d_1, d_2, c, l are positive constants that respectively express the degree of flexibility of the economy, the tolerance threshold of energy supply in relation to energy price and the tolerance threshold of economic growth in relation to energy price and energy supply.

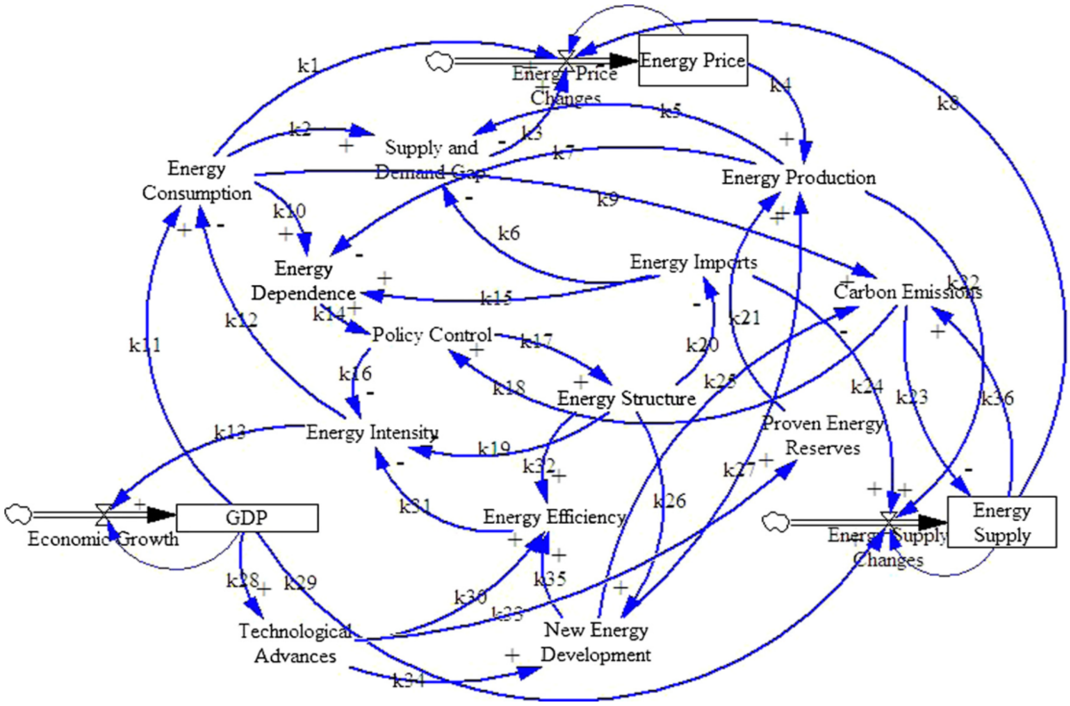

Figure 4. Cause and effect relationships of the system

3.2. Examining model parameters

a_1: The intrinsic growth factor is the price of energy, which by (k_1) shows the effect of energy consumption on energy price ($k_2.k_5.k_6$), respectively shows the effect of energy consumption, energy production, and energy import due to lack of energy supply on the energy market, and (k_3) the effect of a lack of energy supply on energy price approved.

a_2: The effect coefficient of energy changes on the price of energy. By (k_8) the direct effect of energy supply on the price, ($k_1, k_{12}, k_{18}, k_{36}$) confirms the indirect effect of energy supply factors such as carbon emissions, government policies, and energy consumption on the price of energy is confirmed.

a_3: The effect coefficient of economic growth on the price of energy. Since economic growth can increase energy consumption (k_1), and contribute to the advancement of science and technology (k_{28}), the efficiency of using those energies increases (k_{30}) and energy intensity decreases (k_{31}). Therefore, energy consumption is reduced (k_{12}). Changes in energy consumption will lead to energy price fluctuations (k_1). Therefore, the influence factor a_3 is confirmed by the mentioned parameters.

b_1: The emission coefficient of energy supply growth, which is determined by (k_{21}) energy sources that can lead to energy production, (k_{22}) the effect of energy consumption on energy supply, (k_{23}) the effect of carbon emissions on energy supply, and (k_9) the effect of energy consumption Carbon emissions confirmed.

b_2: The impact factor of energy price changes in line with energy supply. In this way, energy price affects energy production (k_4), expands energy production and supply (k_{22}), or reduces energy dependence (k_7). This reduction will lead to regulating energy policies (k_{14}) and then the energy structure (k_{17}). This setting can change energy import (k_{20}) and supply (k_{24}). The stated items confirm b_2.

b_3: The coefficient of economic growth on energy supply. Economic growth can help increase energy consumption (k_{17}), and carbon emissions (k_9) will increase accordingly. Due to the protection of the environment, the increase in carbon emissions prevents the increase in energy production (k_{23}). Economic growth will force the government to adjust its policies (k_{18}) and the energy structure (k_{17}). Energy import (k_{20}) and energy supply (k_{24}) will change. In addition, economic growth will directly increase the exploitation of new energies (k_{34}) and available energy resources (k_{33}). New energies and the development of existing energy sources can promote the production of new energies and energy sources (k_{21}) and (k_{27}) and finally increase the total energy supply. Therefore, the mentioned factors confirm b_3.

$c_1$shows the effect of economic growth on itself. Economic growth will lead to technological progress (k_{28}). Then, it will improve the efficiency of energy use (k_{30}) or the exploitation of new energy, (k_{34}) the exploitation of new energy will change carbon emissions, and (k_{18}) the regulation of policies. Accordingly, the energy structure will change (k_{17}). The two mentioned paths reduce energy intensity ($k_{31}.k_{19}$) and will change the economy's growth (k_{13}). Economic growth increases energy consumption directly (k_{11}) and changes energy dependence (k_{10}), and then policies are set (k_{14}) that lead to energy intensity reduction (k_{16}) and economic growth (k_{13}); therefore, c_1 is confirmed.

c_2: indicates the coefficient of impact of energy supply on economic growth. Energy supply causes changes in carbon emissions (k_{36}) that lead to policy adjustments (k_{18}); these adjustments can help change the energy structure (k_{17}) and energy imports (k_{20}). Based on these cases, energy dependence will change (k_{16}), and the economy will develop (k_{13}). Meanwhile, adjusting the energy structure will increase energy utilization (k_{26}) and help improve energy use efficiency (k_{35}). The goal of reducing energy intensity can be achieved (k_{16}), and therefore, the economy will develop (k_{13}). The stated items are in favor of c_2.

3.3. *Numerical simulation of the model*

The following methods are used to determine the stability of the non-linear dynamic system:

1- Solving the system of differential equations by numerical methods (simulation) and studying the behavior of the solution
2- Linearization of the non-linear system in several points where this method is prone to linearization errors.
3- Defining a Lyapunov function and studying the behavior of the function

In this research, the simulation approach and the ODE45 command in MATLAB, which has high accuracy, have been used.

The economic growth system of energy price and its supply shows the interactive relationships between economic growth, energy price, and energy supply. In the evolutionary process of the system, there are complex dynamic features between the system variables and parameters. We discuss the dynamic characteristics of the system using numerical simulation methods.

Some models have an analytical solution with one or two variables, in which case the values of the variables are easily obtained. However, models with higher dimensions and high complexity that do not have an analytical solution can be analyzed numerically. For this purpose, the model is simulated using MATLAB software and the Simulink toolbox. In order to check the behavior of the system, by selecting the initial parameters and values in Table 1, the simulation is performed with the time horizon of 1000 and the step length of 0.0001, and it is executed with the ODE45 solver that uses the fourth and fifth order Rang Kuta. The numerical solutions obtained by this solver are highly accurate in most cases. Many differential equations cannot be easily solved analytically, but solving them numerically is relatively simple. Hence, a numerical approach is, in general, more applicable.

4. Simulation results

In this section, simulations were performed using MATLAB software. The simulation results are displayed as three-dimensional, two-dimensional, and one-dimensional graphs. The system's behavior has been analyzed for each case.

Table 1. Parameters and initial values of the model for four simulation cases

Case	Z_0	Y_0	X_0	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	d_1	d_2	c	l
1	0.09	0.05	0.8	0.3	0.5563	0.15	0.4	0.6073	0.3	0.3	0.06	1.5	1.5	2.7	1.9
2	0.8	0.2	0.1	0.3	0.5563	0.15	0.4	0.6073	0.3	0.3	0.06	1.5	1.5	2.7	1.9
3	0.9	0.8	0.5	0.3	0.5563	0.15	0.4	0.6073	0.3	0.3	0.06	1.7	1.8	2.7	1.9
4				0.3	0.5563	0.15	0.4	0.6073	0.3	0.3	0.06	1.7	1.8	2.7	1.9

The first case, as depicted in the figure, presents a system exhibiting random behavior. However, this apparent randomness is deceptive. This system is fundamentally deterministic, meaning its future states can be predicted accurately given its mathematical equation and initial conditions. This characteristic classifies it as a chaotic system. Despite the apparent disorder, chaotic systems possess a hidden order that manifests as recurring patterns over time. A crucial aspect of chaotic systems is the sensitivity to initial conditions, often called the "butterfly effect." Even minute changes in starting values can lead to drastically different outcomes. This sensitivity underscores the importance of precise parameter setting in chaotic systems.

Figure 5 further illustrates the quasi-random and chaotic nature of the system. The presence of an attractor, also known as an absorbent area, is evident. This attractor represents a region in the system's phase space where the system's trajectory tends to converge over time, regardless of the initial conditions. The existence of an attractor suggests that while the system exhibits chaotic behavior, it is not entirely unpredictable. The system's dynamics are constrained within the boundaries of the attractor, providing a degree of predictability despite the apparent randomness.

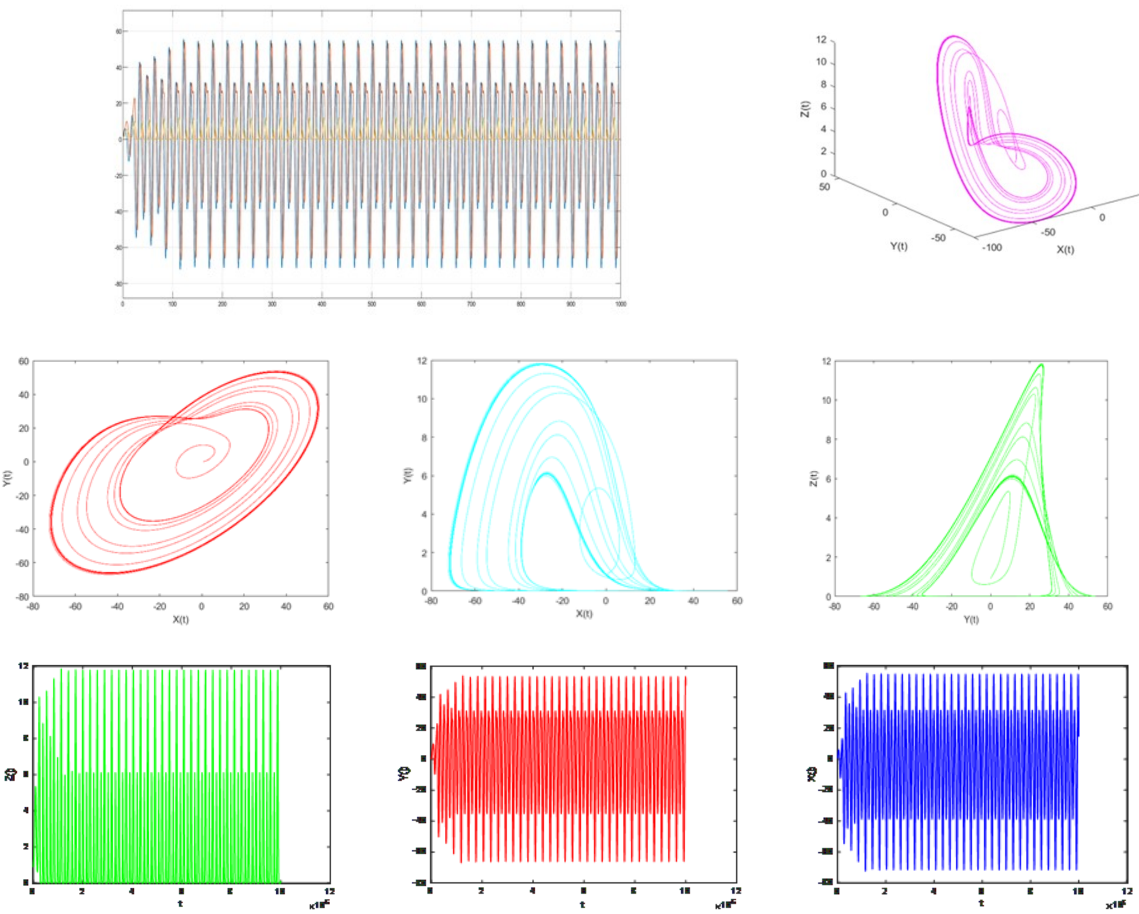

Figure 5. System diagrams for mode 1

 DOI: 10.22067/JSTINP.2024.86349.1089

In the second case depicted in the figure, the system may appear random at first glance. However, it is crucial to note that this system operates deterministically. It implies that possessing the mathematical model of the system at any given moment enables accurate predictions of its future behavior. The system in question exhibits chaotic characteristics, wherein an underlying order emerges amidst apparent disorder and persists over time. A key consideration in chaotic systems lies in the configuration of parameters. Furthermore, Figure 6 illustrates the system's behavior, showcasing a blend of quasi-random and chaotic tendencies. Additionally, the existence of an absorbent area is a notable feature.

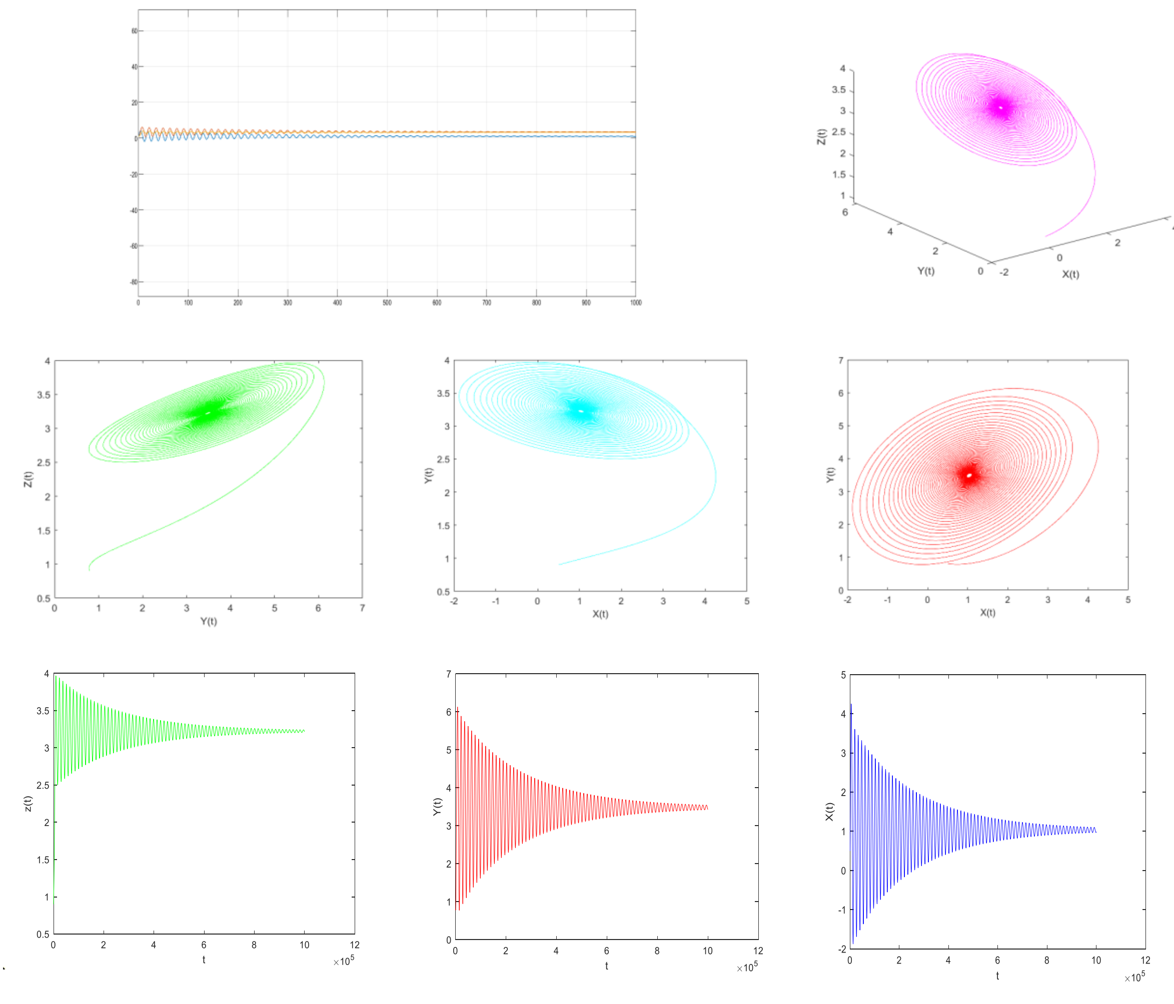

Figure 6. System diagrams for mode 2

In the third case, with the change of the initial values and the fixed values, the system has an oscillating periodicity, and therefore, the system has a limit cycle; in other words, the system has an oscillating cyclic path on which the states of the system are established. The limit cycle of the system can be used in the context of the cyclical economy, which aims to maintain products, compounds, and raw materials at the highest level of usefulness and value over time, to modify and reuse resources, and to distinguish between biological and technical cycles.

Paying attention to this issue can help to recycle and thus reduce greenhouse gas emissions. The behavior of the system in this case is shown in Figure 7.

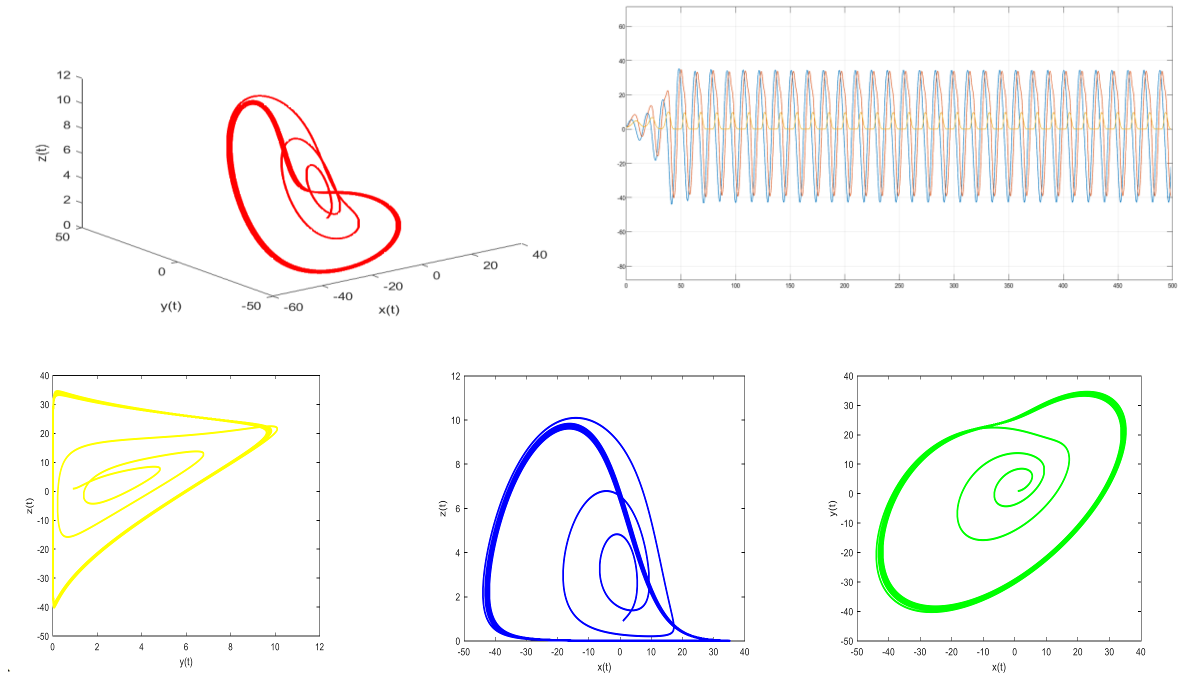

Figure 7. System diagrams for mode 3

In the fourth case, the system eventually converges to a stable equilibrium point where all three state variables stabilize and remain constant over time. This state of equilibrium signifies a balance in the system's dynamics, indicating a point where the system reaches a steady state and ceases to undergo significant changes. The attainment of this stable equilibrium holds significant implications for policy formulation. By understanding and leveraging this balance point, policymakers can design and implement strategies to reduce energy intensity. This reduction can lead to enhanced productivity, decreased environmental impacts such as lower greenhouse gas emissions, and overall improvements in sustainability. Figure 8 visually encapsulates this concept, showcasing the system's progression toward stability and the potential outcomes of achieving a balanced state. Identifying and utilizing such equilibrium points are crucial for guiding decision-making toward more efficient and sustainable energy practices, fostering long-term economic and environmental benefits.

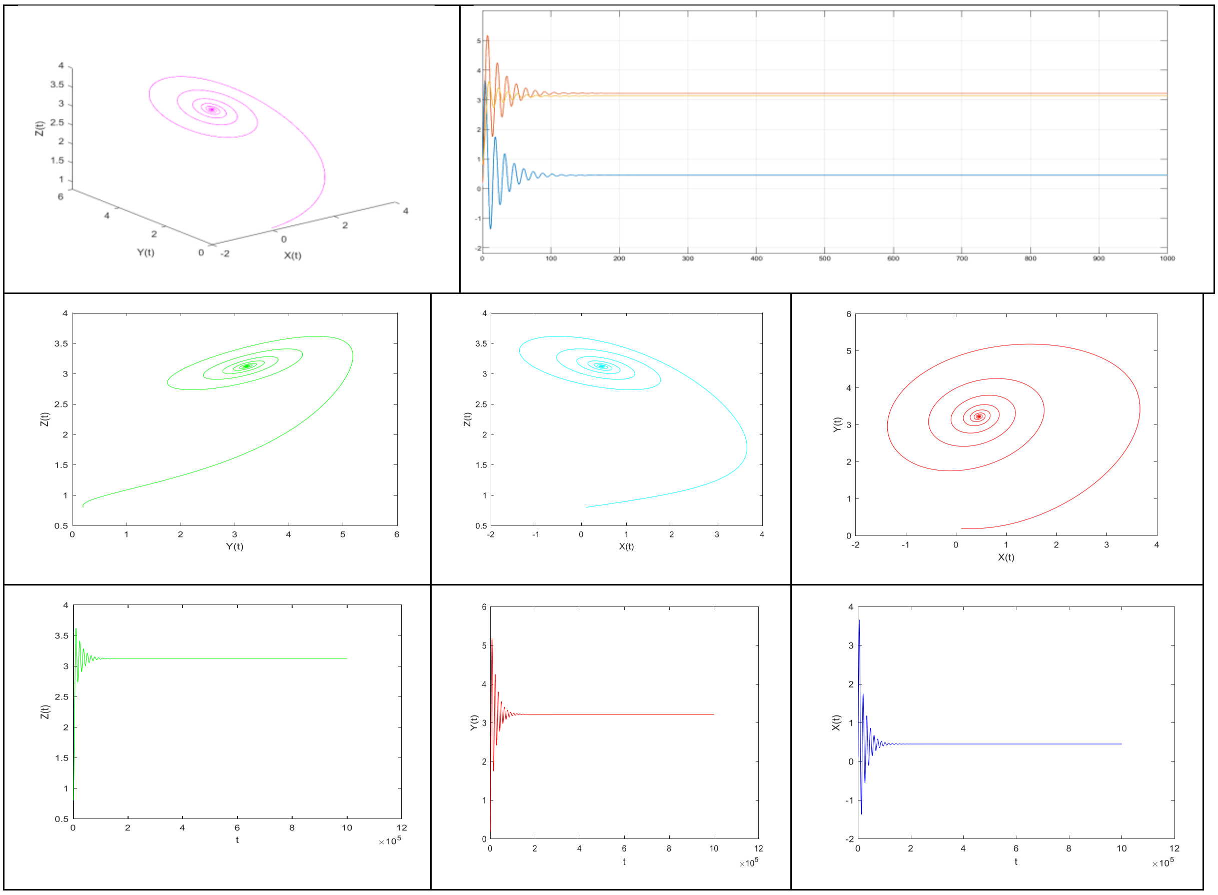

Figure 8. System diagrams for mode 4

Numerical simulation results show a complex nonlinear relationship between energy price, supply, and economic growth. The existing complexity is closely related to the parameters of the model. The model is observed for different values in different periods of the system, which leads to a stable state (Figure 8), intermittent and periodic changes (Figure 7), and a state of disorder and chaos (Figure 5). Also, according to the numerical simulation results, it can be seen that the system shows different states with the same parameters and under different initial conditions, which indicates the system's sensitivity to the initial values.

System Dynamics and Chaos Theory are both tools for studying complex systems. System Dynamics focuses on how systems change over time, especially in long-term, stable systems. Chaos Theory better analyzes systems where small changes lead to big, unpredictable effects. The choice between these approaches depends on the system's characteristics and the research question. Chaos Theory suits systems with non-linearity, emergent properties, and many unknown variables. System Dynamics works better for systems with a detailed understanding of their structure and relationships. However, these approaches can be used together for a more comprehensive understanding of complex systems.

5. Conclusions and suggestions

Most previous research has used economic models regarding energy supply, energy price, and economic growth (Mahadevan and Asafu-Adjaye, 2007; Lee and Chiu, 2011; Tang and Tan, 2013). They have either used empirical analysis (Odhiambo, 2010) (or they have used simple linear and quadratic relationships (Mumuni and Mwimba, 2023; Berk and Yetkiner, 2014). Considering the highly effective factors on the energy supply system, energy price and economic growth, and the complex and non-linear relationships between factors, it is necessary to use the theory of non-linear dynamic systems (chaos) to reduce modeling errors. In this research, after creating a non-linear dynamic model, the execution model and the behavior of the system have been investigated using numerical simulation. Small changes in the initial values change the behavior of the system drastically and the system shows high sensitivity to the initial values. Also; The numerical simulation results show that the chaos of the system is controlled. In other words, the chaotic behavior of the system reached a limit cycle and finally a stable point. Therefore, according to the findings of this research, it is recommended that future studies delve deeper into the theoretical aspects by employing the Lyapunov function to analyze the model's behavior.

Additionally, conducting bifurcation analysis to estimate parameters and establish tolerance thresholds for parameters and constant values within the model should be considered. Practically, utilizing authentic data from Iran's economy is crucial to effectively tackle the intricate challenges within the energy economy and macroeconomic variables. Furthermore, implementing diverse strategies and crafting tailored policies in the energy economy sector, which intersects with environmental, livelihood, and economic concerns, can offer viable solutions to current and future issues in these domains. By developing a novel system tailored to the economic landscape, researchers can derive detailed energy intensity calculation formulas to explore the impact of control intensity on reducing energy intensity across various control strategies. Additionally, conducting quantitative evaluations can provide valuable insights into the effectiveness of control policies in decreasing energy intensity and the time required for implementation, facilitating informed decision-making and policy formulation within the energy sector.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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